INTRODUCTION
Conformance testing is based on measurements performed on pre-selected Key Product Characteristics (KPC), and is a key to ensure product functionality. However, conformance tests are expensive: the overall “uncertainty cost” comes from cost for performing measurement (measurement cost), and cost due to inspection errors (error cost). Unfortunately, measurement cost increase as uncertainty decreases, while inspection errors reduce as uncertainty reduces.

A commonly adopted measurement system for geometric error evaluation is a “Coordinate Measuring Machine” (CMM). Wilhelm et al. have identified [1] several sources of CMMs measurement uncertainty. In particular the sampling strategy can significantly affect measurement uncertainty. In fact, in geometric tolerance verification only those zones of the surface which deviate the most from the design nominal geometry affect the geometric error. It has also been observed that anomalous zones of the part profile/surface tend to be the same throughout a stable production. It may therefore be stated that the part presents a “manufacturing process error signature” (MPES), that is geometric error behavior is similar in every part.

Ceglarek and his team [2] developed methods to model part variation patterns of pre-assembled components to compensate dimensional variability caused by upstream manufacturing processes. In recent years several studies have suggested that the interaction between sampling strategy and manufacturing process error signature can be analyzed in order to generate very effective sampling strategies (e.g. Summerhayes et al. [3], or Colosimo et al. [4]). However, these approaches are currently limited to form error verification. Moreover, the criteria proposed lack a correct economic point of view.

This work proposes a methodology optimizing the sampling strategy when estimating geometric error by using CMM gages. The method is based on a cost function depending on the sampling strategy, so the sampling strategy will be the best trade-off between measurement cost and error cost. An uncertainty evaluation similar to the one in ISO/TS 15530-3 standard is implicit in the methodology. A discussion will be proposed on the impact of lack of tracing ability of CMM gages [5]. An industrial case study will be proposed involving parallelism between planes.

PROPOSED METHODOLOGY
The proposed methodology is based on repeated measurements of few calibrated workpieces. The sampling strategy for these measurements should be as dense as possible, and uniformly spaced, regardless of the considered geometric tolerance. From these dense samplings a pre-selected subset of points is selected, and then a suitable costs function is used to evaluate this strategy. By varying the subset of measurement points, the subset which minimizes the cost function is determined. By applying the methodology to more than one calibrated workpiece, the sampling strategy will tend to concentrate sampling points exactly in those areas which are anomalous (deviate the most from design nominal geometry) throughout the whole production.

The basic structure of the function to evaluate uncertainty costs $C_I$ for a single workpiece can be written as

$$C_I = C_M + C_E$$

where $C_M$ is “measurement cost”, namely, the cost of performing a single measurement task, and $C_E$ is the “errors cost”, that is the cost generated by inspection error. The proposed methods
FIGURE 1. Rejected fraction of conforming produced parts in presence of an USL.

is applicable in cases where $C_E$ is large enough to significantly affect the probability of type I and type II measurement errors.

Evaluation of $C_M$

For a CMM, the cost related to the sampling strategy depends essentially on time involved in taking measurements. Neglecting fixed times for system set-up, measurement time depends on the sample size, so it is possible to express $C_M$ as

$$C_M = c_M t \simeq c_M t_p n = c_p n$$  \hspace{1cm} (2)

where $c_M$ is the CMM hourly cost, $t$ is the time required to perform the measurement task, $t_p$ is the time for sampling a single point, $n$ is the sample size, and then $c_p$ can be interpreted as “cost for sampling a single point”.

Evaluation of $C_E$

The evaluation of $C_E$, depending on the approach chosen by the manufacturer to deal with measurement errors, is quite subjective. In this work, a simple approach is proposed based on the fraction of parts rejected even if conforming.

Suppose a single sided specification limit $SL$ has been defined for some KPC of a workpiece, as depicted in Figure 1. Suppose only an Upper Specification Limit (USL) exist, as usual in geometric tolerances. This means that a part is non conforming if the real value of the KPC $x > USL$. Suppose now that conformance to a tolerance has to be proved, and the ISO 14253-1 standard approach for stating conformance is followed: this standard suggests that a part should be stated non conforming if $y > USL - U$, where $y$ is the measurement result, and $U$ is the expanded uncertainty. Moreover, suppose that $x$, behaves according to some statistical distribution (e.g. a Gaussian distribution). Therefore, if uncertainty increases, a higher number of parts will be rejected, even if they should be accepted (Figure 1). $C_E$ may then be evaluated as

$$C_E = c_w P(USL - U < x \leq USL)$$  \hspace{1cm} (3)

where $c_w$ is the cost of type I error, $P(USL - U < x \leq USL)$ can be regarded as the probability that the real geometric error falls between $USL - U$ and $USL$, so we cannot state conformance for the part, but the part is conforming. Therefore, this probability represents the average fraction of conforming parts declared non conforming. Usually, only an upper bound exists for a geometric tolerance, so Eq. (3) considers only an upper bound. However if both upper and lower bound exist, Eq. (3) can be easily modified.

This expression of error cost neglects parts which are considered conforming even if they do non conform, which come with a cost, of course. In fact, the probability that this happens if ISO 14253-1 rule is followed is very low.

Uncertainty evaluation

The ISO/TS 15530-3 technical specification proposes a procedure to evaluate measurement uncertainty which is based on raw data obtained from repeated measurements of a calibrated artifact. From raw data, some terms are estimated, such as: $u_{cal}$ (uncertainty contribution due to calibration uncertainty), $u_p$ (uncertainty contribution due to measurement procedure), $u_W$ (uncertainty contribution due to variability of the manufacturing process), and $b$ (measurement bias). The term $U$ (expanded uncertainty, see GUM and VIM) is evaluated as

$$U = k \sqrt{u_{cal}^2 + u_p^2 + u_W^2 + |b|}$$  \hspace{1cm} (4)

where $k$ is the expansion factor. To evaluate the influence of the manufacturing signature on the uncertainty, more than a calibrated part as to be adopted. However formulas in ISO/TS 15530-3 are not suitable for this. Therefore, a modification of the standard is proposed, so that more than one calibrated artifact may be used. In particular, the $b$ term (the average bias) should be evaluated as

$$b = \frac{\sum_{j=1}^{m} \sum_{i=1}^{r_m} (y_{i,j} - x_{cal,j})}{mr_m}$$  \hspace{1cm} (5)

In Eq. (5) $m$ is the number of calibrated artifacts adopted, $r_m$ is the measurement repetitions num-
ber for each artifact, \( y_{i;j} \) is the measurement result (estimated geometric error) of the \( i^{th} \) measurement repetition of the \( j^{th} \) artifact, and \( x_{\text{cal};j} \) is the reference value for the \( j^{th} \) artifact (calibrated geometric error). It is supposed that each calibrated workpiece is measured the same number of times; to be as similar to ISO 15330-3 standard as possible, it is suggested \( r_m \geq 10 \). Then, to estimate \( u_p \), a pooled standard deviation could be used:

\[
 u_p = \sqrt{\frac{\sum_{j=1}^{r_m} \sum_{i=1}^{n_m} (y_{i;j} - \bar{y}_j)^2}{m(n_m - 1)}} \quad \bar{y}_j = \frac{\sum_{i=1}^{n_m} y_{i;j}}{r_m} \tag{6}
\]

Substituting Eq. (5) and (6) in Eq. (4) an evaluation of \( U \) which takes into account more than one calibrated artifact, and then the interaction between the sampling strategy and the MPES, is obtained.

**Strategy optimization**

Having identified a methodology to evaluate uncertainty based on raw data, the next step is choosing a sampling strategy that minimizes the uncertainty cost. The solution of the problem is not straightforward. Suppose that \( r_m \) dense measurements of \( m \) calibrated parts have been obtained, and that the sampling strategy is the same for every measurement. To solve the minimization problem, the sampling points corresponding to any sampling strategy may be extracted from these clouds of points. The extracted subsets of points can be introduced in the measurement uncertainty estimation procedure. If the sampling pattern is effective, i.e., it is able to detect regions of the feature that deviate the most, then uncertainty will be low. The identification of an optimal pattern can be seen as an optimization problem where at most any different alternative pattern is compared. However, due to the combinatorial nature of the problem, it is not possible to consider any strategy. Therefore, genetic algorithms or simulated annealing algorithms should be adopted for optimal strategy definition, given the discrete nature of the problem.

Finally, even if the optimization of the strategy is based on the presence of a MPES, explicit knowledge of the signature is not required for the optimization itself.

**CASE STUDY**

As a case study, the parallelism defined in point (e) of Table 3 in the ISO 10791-7 standard is considered. Ten parts were milled and a 0.045 mm parallelism tolerance was defined. The manufacturing cost was evaluated in \( 40 \), that is taken as \( c_W \). Then tolerances and datum features were sampled by means of a CMM, on a uniformly spaced sampling strategy, with a point density of 1 point/mm\(^2\); a total of 3720 points were sampled on each part (1395 on the datum feature, and 2325 on the toleranced feature). Measurement was repeated ten times for each part. Finally, every part was calibrated with a standard calibration uncertainty \( u_{\text{cal}} = 0.001 \) mm. From calibrated parallelism errors, it was ascertained that parallelism for these parts was distributed according to a \( N(0.0402; 0.019) \) Gaussian statistical distribution (mean and standard uncertainty expressed in [mm]). The average surface of the ten toleranced surfaces is plotted in Figure 2: it is apparent that the surface presents a sawtooth profile and a trend along the \( y \) axis.

Finally, a simulated annealing algorithm was applied in order to select an optimal sampling strategy. Please note a parallelism is being considered, so sample size sums both the points sampled on the toleranced feature and the corresponding datum, and, throughout the optimization process, sampling points are left free to “migrate” from datum to toleranced feature and vice versa. Optimal strategy was compared to a standard Hammersley strategy. Figure 3 shows the behavior of expanded uncertainty \( (k = 2, u_W = 0) \) as the sample size varies. As expected, as the sample size increases uncertainty reduces; it is apparent that the proposed strategy greatly outperforms Hammersley strategy. Therefore, the proposed strategy should be useful even if the systematic error b term is compensated.
Figure 4 shows the relationship between uncertainty cost and sample size. Non monotonic behavior is due to that initially uncertainty is large, so $C_E$ is large, then uncertainty quickly drops, and then cost reduce. However, as the sample size increases uncertainty improvement does not compensate anymore for sample size increase, so $C_I$ tends to increase. Anyway, because of lower uncertainty the proposed strategy always show lower cost.

Effect of the lack of tracing ability
The 'lack of tracing ability' means that instead of measuring the given sampling points on the feature, the CMM may actually measure the area around them [5]. The lack of tracing ability may be due to several factors including alignment errors, varying slope of the measured surface, and dimensional variability of parts. This limitation to the capabilities of coordinate measuring systems generates an additional source of uncertainty, which tends to inflate the measurement procedure uncertainty.

This can be a real problem for signature based strategies. In fact, the ability of this kind of strategies to generate optimal results relies on its ability to measure exactly a particular point on a surface. In fact, Figures 3-4 show that a shift in the same direction of all sampling points equal to 1 mm is sufficient to greatly reduce performance of the proposed strategy. Therefore, if adopting a signature optimized sampling strategy, great care should be taken to ensure that parts are correctly traced.

CONCLUSIONS
In this paper a methodology has been proposed to be able to plan sampling strategies for inspecting geometric tolerances. The generated strategy optimizes uncertainty cost. Uncertainty cost is mainly linked to the number of sampling points taken, and the probability of errors. The optimization of the sampling points locations is based on the presence of a "manufacturing signature", that is a systematic behavior of the real geometric feature. The proposed methodology suggests the measurement of uncertainty evaluation which takes into account the interaction between the sampling strategy as well as manufacturing signature. Therefore, though the comparison of several possible sampling strategies, the one characterized by the optimal interaction may be selected.

REFERENCES