ANALYSIS OF BALLSCREW STIFFNESS OWING TO CONTACT DEFORMATION IN LEADS CREEW SYSTEMS

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ABSTRACT
Positioning error of ballscrew driven feeddrives depends upon stiffness of leadscrew systems. Total axial stiffness of the leadscrew system consists of stiffnesses of the nut, the screw shaft, the support bearings, and the nut bracket. The nut stiffness depends upon screw characteristics, preload and operating conditions. This should be thoroughly studied according to the contact deformation between balls and grooves. In this paper, ballscrew nut stiffness due to contact deformation between balls and grooves according to the preload and operating conditions is studied. Positioning accuracy estimation method is also proposed.

KEYWORDS: Ballscrew, Contact deformation, Hertzian constant, Positioning error, Stiffness

INTRODUCTION
Ballscrews utilize rolling contact with low friction. They are widely used as precision feeddrive units of machine tools, flat-panel fabrication devices and semiconductor manufacturing equipment. Positioning error of the ballscrew driven unit depends upon geometric error and stiffness of the leadscrew system. Stiffness estimation of the ball screw is critical for accurate positioning and design of precision machinery.

The nut stiffness depends upon screw characteristics, preload and operating conditions. Recalling that the contact deformation of the nut assembly governed by Hertzian contact is affected by the groove shape, preload and thrust force, the nut stiffness of ballscrews should be thoroughly studied according to the contact deformation between the ball and groove. For stiffness calculation, a rigid model assuming the nut and screw shaft are rigid except for contact points with balls had been used. However, it has been pointed out that the measured stiffnesses are considerably lower than the calculated values obtained from the rigid models [1,2]. They claimed that the calculated stiffness incorporating the flexibility of the screw shaft and nut matched the measured stiffness. However, they did not present explicit theoretical expressions for the stiffnesses of ballscrews. In this paper, calculation of nut stiffness due to the contact deformation between balls and raceway grooves incorporating flexibility of the ballscrew shaft and nut is studied. The estimation of the stiffness and positioning error due to the contact deformation is conducted as well.

HERTZIAN CONSTANT
Fig. 1(a) shows a cross section of a ballscrew composed of balls, shaft, nut and circular arcs in TC (Tension in the shaft and Compression in the nut: forward motion case) loading condition. Contact state of a ball and grooves is shown in Fig. 1(b). D means the ball diameter, α is the contact angle of the ball and grooves, γ represents the lead angle of the screw shaft, P and Pi stands for contact loads, Fa is the thrust force applied, and fn and fs denote conformity factors of the screw and nut grooves, respectively.
Considering geometry at contact points shown in Fig. 1(b), curvatures on the elliptic contact surfaces in normal axes x and y are obtained \[4\]. Solving the first and second kinds of elliptic integrals through the Landen transformation and approximation when the contact load \(P\) is applied as shown in Fig. 1(b), contact deformations \(\delta^H_{bn}\) between the ball and nut groove, and \(\delta^H_{bs}\) between the ball and shaft groove are given by

\[
\delta^H_{bn,bs} = \frac{\rho_{bn,bs}}{2} \left[ \frac{3P}{2} \left( \frac{(1 - \zeta_b^2) + (1 - \zeta_{ns,b}^2)}{E_b} \right) \right]^{\frac{3}{2}}
\]

where \(\delta^H_{bn,bs}\) means \(\delta^H_{bn}\) and \(\delta^H_{bs}\). \(\delta^H_{bn}\) is computed from \(\delta^H_{bn}\), \(\rho_{bn}\), Young's modulus of nut \(E_n\) and Poisson ratio of nut \(\zeta_n\). According to Hertz contact theory \[4\], contact deformations due to load \(P\) between the ball and raceway groove are given by

\[
\delta^H_{bn,bs} = C_{bn,bs} P^3
\]

Comparing Eqs. (1) and (2), Hertzian constants are given by

\[
C_{bn,bs} = \frac{3}{2} \left[ \frac{\rho_{bn,bs}}{E_b} \left( \frac{(1 - \zeta_b^2) + (1 - \zeta_{ns,b}^2)}{E_b} \right) \right]^{\frac{2}{3}}
\]

These constants are determined according to curvatures of the contact points, conformity factors of the groove arch, ballscrew geometry, material property, etc.

\[
P = \frac{F_p}{Z \sin \alpha \cos \gamma}
\]

where \(Z\) is total number of loaded balls, and \(\alpha\) and \(\gamma\) are the contact and helix angles of the ballscrew, respectively. Total axial contact deformation due to the preload is

\[
\delta^H_p = \left( \frac{C_{bn} + C_{bs}}{\sin \alpha \cos \gamma} \right)^{\frac{2}{3}}
\]

where Hertzian constant of the ballscrew \(C_b\) is given by

\[
C_b = \left( \frac{C_{bn} + C_{bs}}{Z} \right)^{\frac{2}{3}} \left[ \sin \alpha \cos \gamma \right]^{\frac{2}{3}}
\]

Figure 3 and 4 show Hertzian constant according to the conformity factor and contact angle of the ballscrew assembly given in Table 1. As the conformity factor expresses the ratio of the curvature radius of the screw groove and the radius of the ball, groove type varies the conformity factor and contact angle. These values affect the variation of Hertzian constant significantly. For the constant conformity, Fig. 4 shows the axial stiffness degraded seriously when the contact angle is less than 30°. It is confirmed that ballscrew stiffness is severely influenced by the groove shape of the ballscrew.
Ballscrews are usually used as a preloaded double nut to eliminate backlash and to improve the axial stiffness. Eq. (5) gives axial deformations of preloaded nuts shown in Fig. 2 in opposite direction (TT mode: Tension in shaft and Tension in nut; backward motion case). Since the preloaded deformations are canceled each other, positioning error due to the contact and elastic deformation does not generate when the thrust force is not applied to the screw assembly. As the thrust force $F_a$ is applied to the preloaded ballscrew as shown in Fig. 2, ball load increases in the working nut and decreases in the preloaded nut. Since the increment of the axial deformation in the working nut equals to the decrement of it in the preloaded nut, following nonlinear equations are obtained for $0 \leq F_a \leq 2.82F_p^*$:

$$P_a^3 + P_b^3 = 2\left[\frac{F_p}{Z \sin \alpha \cos \gamma_a}\right]^3 \quad (7)$$

where $P_A$ and $P_B$ are uniformly distributed ball load due to nut load $F_A$ and $F_B$ developed by the thrust force $F_a$ and preload $F_p$. After solving $P_A$ from Eqs. (7) and (8), net axial deformation due to the contact load is obtained as

$$\delta_a^H = \delta_a^H - \delta_p^H = \frac{C}{\sin \alpha \cos \gamma_a} \left[\frac{P_A}{Z}\right]^3 - \frac{C}{\sin \alpha \cos \gamma_a} \left[\frac{F_p}{Z}\right]^3 \quad (9)$$

where $C = (C_{bn} + C_{bn})$. Fig. 5 shows elongation errors due to the axial contact deformation according to the thrust force $F_a$ when $F_p = 0$ (nonpreload) case and $F_p = 300$ Kgf, respectively. It is confirmed that the stiffness is improved significantly by incorporating the preload to the ballscrew.

In addition to the Hertzian contact deformation, the axial deformation error of the ballscrew is generated from elastic deformation of the screw shaft and the nut body as well. On the $i$th and $(i-1)$th ball locations, ball loads $P_i$, $P_{i-1}$, and shaft deformation $\delta_s^i$, $\delta_s^{i-1}$ and nut deformation $\delta_n^i$, $\delta_n^{i-1}$ are defined in the ball contact direction. Using the compatibility and force equilibrium conditions, following body deformations between the two balls have been obtained:

$$\delta_{s,i}^i = \frac{L/2}{E_s \cdot A_s} \left[\left(P_i \sin \alpha_{i-1} - P_i \sin \alpha_i\right) \cdot \cos \gamma\right]$$

$$\delta_{n,i}^i = \frac{L/2}{E_n \cdot A_n} \left[\left(P_i \sin \alpha_{i-1} - P_i \sin \alpha_i\right) \cdot \cos \gamma\right]$$

$$\delta_{s,i}^{i-1} = \delta_{s,i}^i + \delta_{s,i}^i$$

$$\delta_{n,i}^{i-1} = \delta_{n,i}^i + \delta_{n,i}^i$$

where $A_n$, $A_s$ are effective cross sectional areas of the screw shaft and nut, respectively. $L$ is the distance between uniformly selected two balls. For the loading condition TT shown in Fig. 2, total body deformation is

$$\delta_{total} = \delta_{s,i}^i + \delta_{n,i}^i$$

From Eqs. (9) and (13), total axial deformation of the ballscrew is given by

$$\delta_{total} = \delta_a^H + \delta_b^B \quad (14)$$

For a longer nut length ($Z=84$ comparing with 62) ballscrew similar to Table 1, Fig. 6 shows axial
deformations according to the thrust force. Axial deformation due to Hertzian contact deformation in TC and TT loading conditions are much the same. Adding the body deformation to the axial deformation, total axial deformations of TC case is larger than those of TT case. It is confirmed that both cases have larger deformation errors than those of Hertzian deformation case. Fig. 7 shows axial stiffnesses according to thrust forces. TT case has larger stiffness than TC case. Both stiffnesses are less than those of the Hertzian deformation case. As the ball screw moves in forward (TT case) and backward (TC case) directions during operation, stiffness variation generates positioning errors of the precision machinery. To improve stiffness and reduce stiffness variation during motion, preload should be applied to the ball screw, as well as groove shape for manipulation of conformity factor and contact angle, ball size, and nut length and dimensions should be designed and selected properly.

POSITIONING ERROR ESTIMATION
To fabricate precision machinery, estimation of positioning errors of ball screw driven feeddrives are available and amendable in the design process accurately. As the positioning accuracy depends upon geometric errors and stiffness of the lead screw system, the estimated axial positioning error is modeled as

$$\delta_{\text{ext}} = \delta_{\text{total}} + \delta_{\text{grade}}$$ (15)

where $\delta_{\text{grade}}$ is the accumulated reference lead deviation of the ball screw, and $\delta_{\text{total}}$ is given by Eq. (14) according to the preload $F_p$, axial load $F_a$, and the ball screw assembly conditions described in previous sections. $\delta_{\text{grade}}$ is selected from the performance certificate given by the ball screw manufacturer. Using Eq. (15), designers are able to predict the positioning accuracy of their precision machinery in the design process. They can select effective ball screws from the market confidently.

CONCLUSIONS
Mathematical modeling of ball screw stiffness has been conducted through the contact and body deformation analysis in various loading and assembly conditions. It is confirmed that the stiffness depends upon ball screw groove geometry, as well as ball, nut and shaft dimensions, and loading conditions. During operation of the precision machinery, the stiffness varies according to the geometric error and the moving direction. This degrades the positioning accuracy. To predict the positioning accuracy and feedback it to the design process of the feeddrive unit, the mathematical estimation method of positioning errors has been conducted.

REFERENCES