1. Introduction

The frequency ramped continuous wave (FMCW) technique for distance ranging originated as a radar technique. More recently, its optical counterpart has been developed for absolute laser interferometry. However, a major disadvantage of this technique has been its lack of accuracy, due to the limited chirp rates available with the laser sources that have been used. The most commonly used source has been a semiconductor laser diode, tuned by varying its injection current. In this case, the sweep range is limited by mode hopping, and the sweep rate is limited by thermal effects. Mechanically tuned external cavity lasers have also been used as sources. These have much greater sweep ranges, but because the tuning is mechanical, their sweep rates are slow. Here, the use of acoustooptically tuned external cavity lasers is proposed, since these have the potential of sweeping rapidly over wide tuning ranges without mode hops. The resulting performance gains are estimated, and a brief description of a possible design is outlined. (A paper describing the design and some preliminary proof of concept tests is published elsewhere.)

The principle of the FMCW method is to mix the light beam returned from the target reflector with the beam emitted by the source: the frequency of the resulting beat frequency is then a measure of the distance ($x$) between the source and the target. If the speed of light is $c$ and the chirp rate of the beam is $\nu$, then the beat frequency is

$$f = \frac{2ix}{c}. \quad (1)$$

Kubota et al [1] proposed and demonstrated an early version of such a scheme, which had a range of a few metres, limited by the coherence length of the source. This was a $0.79 \, \mu\text{m}$ (380 THz) laser diode, and its injection current was varied in order to generate a frequency ramped output beam: the modulation of optical frequency was about $\pm30$ GHz at a modulation rate of 90 Hz. This corresponds to a chirp rate of about 2.7 THz/s. At a range of 1 m, the beat frequency would therefore have been 18 kHz.

More recently, Minoni et al [2] reported on a low cost, compact and robust implementation of a system based on the same principles. In this case, the beat frequency was about 52 kHz for every metre of range. They reported that the frequency measurement resolution was 3.5 Hz, giving a spatial resolution of about 70 $\mu$m.
Both these systems used semiconductor laser diodes as sources, chirped by varying the injection current. This type of chirped source has two disadvantages. Firstly, the optical frequency can be modulated by no more than a few tens of GHz before mode hopping occurs (and the effect of a mode hop would be catastrophic). Secondly, the modulation rate is limited to no more than a few hundred Hz by the thermal time constant of the diode. As a result, the chirp rate can be no more than about 10 THz/s, so the beat frequency can be no more than a few tens of kHz at a range of 1 m. To increase this beat frequency, a wider range of modulation frequency and/or a faster modulation rate are required to generate a faster chirp.

The mechanically tuned Littman design [3] is well established as the basis for many phase continuously tunable external cavity lasers. (The phrases “phase continuously tunable” and “mode hop free” are synonymous.) Its key attribute is that it has a wide tuning range: Stone et al [5] used a laser of this type as the source for an absolute interferometer. Although the tuning range can be very wide, the sweep speed is limited by the mechanical drive. For example, a commercial product [4] can tune, free of mode hops, from 1440 to 1640 nm, an optical frequency range of 25 THz. However, the mechanical tuning limits the maximum sweep speed to 80 nm/s, which corresponds to a chirp rate of 10 THz/s. (Incidentally, the device used by Stone et al had a sweep speed of only 8 nm/s.)

The elimination of moving parts from such an external cavity laser would overcome the sweep speed restriction, and in addition would remove motor vibration and increase alignment stability. Acoustooptic tuning is an attractive way of increasing the sweep speed by at least two orders of magnitude. Moreover, the lasing wavelength is controlled electronically, allowing chirps of very high linearity to be generated.

Although acoustooptic tuning of lasers is well established, mode hop free (continuous phase) tuning over wide frequency ranges has not in the past been achieved. However, a scheme for doing this has recently been proposed, and its feasibility has been demonstrated experimentally [6]. This type of source is capable of ramping at 1000 THz/s or more, and could significantly improve the performance of absolute interferometers.

This abstract is organized as follows: Section 2 discusses the relationship between chirp rate and accuracy; Section 3 describes how an interferometer of this type could be used not only for ranging, but tracking a moving object; Section 4 summarizes the principle of the acoustooptically chirped laser.

2. Accuracy

From equation (1) it follows that the error in range $\Delta x$ resulting from an error in beat frequency measurement $\Delta f$ is

$$\Delta x = \frac{c}{2\nu} \Delta f.$$  

(2)
The accuracy with which $\Delta f$ can be measured varies inversely with the measurement time $T$ (which may extend over more than one frequency sweep). It is also to be expected that the better the signal to noise ratio $R$ of the beat signal, the more accurately can its frequency be measured. An approximate estimate derived in the Appendix suggests that

$$
\Delta f = \frac{1}{2TR^{1/3}}.
$$

(3)

As an example, assume a source with a chirp rate of 5 THz/s, a measurement time of 10 ms and a signal to noise ratio (SNR) of 125. Then the accuracy with which the beat frequency can be measured is 2 Hz, and the range accuracy is 60 $\mu$m.

Now assume a source with a chirp rate of 500 THz/s. The bandwidth of the electronics used to measure the beat frequency will now need to be increased by a factor of 100, and it is assumed that consequently, the noise will increase by a factor of 10, reducing the SNR to 12.5. Again assuming a measurement time of 10 ms, the beat frequency can be measured to about 9 Hz, and the range to about 3 $\mu$m.

These examples are fictitious, and they depend on a number of assumptions. Nevertheless, they do indicate that significant range accuracy improvements could be obtained by using faster chirp rates.

3. Tracking a moving object

So far, it has been assumed that the interferometer is used for ranging a stationary object, and equation (1) is written on that basis. However, it is, at least in principle, possible to use this system for tracking a moving object. Imagine that the object is moving away from the source at a velocity $\dot{x}$. Then a Doppler shift term must be added to the right side of equation (1), which becomes

$$
f = \frac{2i\dot{x}}{c} + \frac{2xv}{c} = \frac{2}{c} \frac{d}{dt}(i\nu) \quad \text{(5)}
$$

where $t$ represents time. Now assume that at a time $t_1$, the instantaneous optical frequency is $\nu_1$ and that the range is known to be $x_1$. At a later time $t_2$, the optical frequency is $\nu_2$. During this time, $N_{12}$ beat cycles have occurred. By integrating equation (5), it can be shown that the range at $t_2$ is given by

$$
x_2 = \frac{c}{2} \frac{N_{12}x_1 + x_1\nu_1}{\nu_2}.
$$

(6)
To avoid a sign ambiguity in frequency when the object is moving towards the source, its speed must not exceed $x \dot{v} / v$. At a range of 1 metre, a frequency of $200 \text{ THz}$ and a chirp rate of $500 \text{ THz/s}$, this critical speed is $2.5 \text{ m/s}$.

4. Acoustooptic synchronously tuned laser

A vital attribute of the Littman laser design is its ability to tune without discontinuities of phase. A particular geometrical configuration is adopted which ensures that as the cavity is tuned by changing its length, the pass band of the mode filter is tuned in exact synchronism with the cavity resonance. It is this synchronous tuning of the lasing mode and the mode filter that allows phase continuous tuning to be obtained over large frequency ranges.

Until now, no corresponding form of synchronous tuning has been demonstrated for acoustooptically tuned lasers. However, a simple condition for synchronous tuning in such devices has now been derived. The cavity in the figure below is designed on this basis. The tuning elements are the two acoustooptic devices, one configured as an upshifter and the other as a downshifter. To generate a linear chirp, these are driven by two acoustic frequency ramps, and it can be shown that if the time separation between these ramps is chosen correctly, synchronous tuning will result. (The required time separation is equal to one half of the product of the cavity’s round trip transit time and the rate of change of the mode filter’s optical pass frequency with acoustic frequency.)

![Acoustooptic synchronous tuning diagram](image)

An acoustooptically tuned laser design which is capable of phase continuous tuning over wide frequency ranges. Gain is provided by the semiconductor optical amplifier (SOA) and the cavity is tuned by the acoustooptic deflectors (labeled “upshifter” and “downshifter”) in conjunction with the diffraction gratings. The RF inputs to the deflectors ($f_1$ and $f_2$) are both ramps, but slightly displaced in time. When this time displacement is appropriately chosen the laser emits a phase
continuous chirp over a wide frequency range. The output is derived from the specular reflection at the first grating. The waveplates simplify the mechanical design by allowing the laser to be constructed on a flat baseplate.

This particular design has not been built, but a simpler version has been built and successfully tested to prove the principle of operation. In particular, it tuned synchronously in the near IR (about 1550 nm). Further details are given in reference [6].

5. Summary

The principle of FMCW absolute interferometry is well established. However, the accuracy of this technique has so far been limited by the chirp rate of the sources available. A significant improvement could be made by using a continuously tunable (mode hop free) acoustooptically tuned external cavity laser as the source. The chirp produced by this type of source can be made highly linear, which also improves accuracy. The FMCW technique is capable not only of ranging stationary objects, but also of tracking moving ones.

Appendix

Assume that the sinusoidal signal whose frequency is to be measured can be represented as \( V_1 = \cos 2\pi ft + n(t) \) where the second term represents noise. Define the signal to noise ratio, \( R \), as the reciprocal of the root mean square of the noise term, that is \( R = \left( \langle n^2 (t) \rangle \right)^{-1/2} \). The SNR is assumed to be significantly greater than 1. A simple method of measuring the frequency would be to count the number of zero crossings in a time \( T \). However, the precision of this method would be limited because only complete half cycles were being counted.

Better precision could be obtained by frequency doubling the signal before counting the number of zero crossings. Successive frequency doublings would further increase precision. However, each time the frequency is doubled, the signal to noise ratio is reduced, and eventually it will fall to below 1. At this point, it is assumed that there is no point in carrying out further doublings. If the number of doublings carried out is \( N \), then the precision of the frequency measurement will have been increased by the factor \( 2^N \).

Assume that frequency doubling is accomplished by squaring the signal. Thus, the original signal becomes

\[
V_1^2 = \left[ \cos 2\pi ft + n(t) \right]^2 = \frac{1}{2} + \frac{1}{2} \cos 2\pi (2ft) + 2n(t) \cos 2\pi ft + n^2(t)
\]

The second term on the right is the new signal at double the frequency of the original, and its amplitude is half that of the undoubled signal.
The third and fourth terms represent noise. It is assumed that the third term dominates the fourth one, which is therefore ignored. (This will be valid only if the signal to noise ratio is significantly greater than 1. However, since the doubling process is truncated when the ratio is equal to 1, the error introduced is small.) The root mean square of the third term is

\[
\langle 4n^2(t) \cos^2 2\pi f t \rangle^{1/2} = \langle 4n^2(t) \left[ \frac{1}{2} + \frac{1}{2} \cos 2\pi (2f t) \right] \rangle^{1/2} = \sqrt{2} \langle n^2(t) \rangle^{1/2}.
\]

Thus, every time the sinusoid is doubled, the amplitude of the signal is halved, and the noise is increased by a factor of \( \sqrt{2} \): thus the SNR is decreased by the factor \( 2\sqrt{2} \). Hence, the number of doublings required to reduce the SNR of the original signal (which was \( R \)) to 1 is equal to \( N = \frac{1}{2} \log_2 R \). The corresponding frequency is

\[
f \times 2^N = f \times R^{\frac{1}{2\sqrt{2}}}.\]

In the measurement time \( T \), the number of zero crossings of a sinusoid of this frequency is \( 2fTR^{\frac{1}{2\sqrt{2}}} \). The relative uncertainty in measuring this frequency by counting zero crossings is \( \left( \frac{1}{2} fTR^{\frac{1}{2\sqrt{2}}} \right)^{-1} \) and so the frequency error is \( \Delta f = \left( 2TR^{\frac{1}{2\sqrt{2}}} \right)^{-1} \).

References