A SYSTEMS APPROACH TO MODELING PIEZORESISTIVE MEMS SENSORS

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SYNOPSIS
Piezoresistive sensing systems have characteristics that enable them to act as fine-resolution, high-speed force and displacement sensors within MEMS. High-performance piezoresistive sensing systems are often difficult to design due to tradeoffs between performance requirements, e.g. range, power, bandwidth, and footprint. Given the complexity of the tradeoffs, traditional approaches to system design have primarily focused upon optimizing a few, rather than all, elements of the sensing system. This approach leads to designs that underperform, often by a significant margin. In this paper, we present a model that captures all significant noise sources, thereby enabling a systematic approach to the design of piezoresistive sensor systems. Improvements can be made in the performance of piezoresistive MEMS sensors using the knowledge of the tradeoffs between design parameters.

INTRODUCTION
Piezoresistors are widely used in MEMS sensor systems due to their low cost, small size, low phase lag, and large dynamic range. They have been used to create MEMs nanomanipulators [1], biocharacterization instruments [2], pressure sensors [3], inertial sensors [4], mass sensors [5], and elements of high-speed atomic force microscopes (AFMs) [5,6,7]. One must understand and manage the tradeoffs between size, bandwidth, resolution, power, and dynamic range in order to obtain high-performance in piezoresistor-based sensor systems. This requires the ability to accurately predict noise sources and how their effects propagate through a sensing system. In this paper we present a systems approach to understanding and managing dominant noise sources.

A thorough study of noise sources adds certainty to the design process, and can yield systems that have sensing which is competitive with capacitive and optical sensor systems. For MEMs applications, this means that one can use cost- and size-appropriate sensors. For example, AFM systems with piezoresistive cantilevers are smaller, have better sensitivity and higher bandwidth than those which use photoelectric sensors [5].

The three most common types of MEMs piezoresistors are single crystal silicon, polysilicon and metal film piezoresistors. Single crystal silicon piezoresistors typically have the highest dynamic range due to their high gauge factors (20 to 100 depending on doping concentration [8, 9]) and low flicker noise. The gauge factor of single crystal silicon depends upon crystallographic orientation [10], therefore this material is typically only used in single axis, cantilever-type force sensors [6, 8, 9]. For multi-axis devices, polysilicon and metal piezoresistors are typically used given the gauge factor is fairly isotropic. [11]. Polysilicon piezoresistors tend to have a lower gauge factor (10-40 depending on doping [10]) and higher flicker noise than single crystal silicon due to the effect of grain boundaries [12, 13]. Metal film piezoresistors have a significantly lower gauge factor (~2) than single crystal and polysilicon piezoresistors but also have lower flicker noise due to their higher carrier concentration [5].

PIEZORESISTIVE SENSOR SYSTEM MODEL

System Architecture
We have created a model that describes the noise dependent performance of the DC piezoresistive sensor system that is depicted in Fig. 1. This model is generic in the sense that the sensor may be used to measure a force, $F$, that is applied to the compliant element or a displacement, $\delta$, of the compliant element. The sensor system is composed of a voltage source, $V_s$, which energizes a span temperature compensated (STC) Wheatstone bridge. An instrumentation amplifier boosts the signal from the bridge, which is nulled with a bias voltage and read by an Analog-to-Digital Converter.
(ADC). This circuit is commonly used and adds little noise to the strain signal.

FIGURE 1: Schematic layout of DC piezoresistive sensor system.

Model Form
Thermal, electrical and mechanical noise sources are incorporated into the model as either measured or predicted values. Partial derivatives of the full model yield the sensitivity of the signal output, $\Psi_M$, to noise sources. The noise spectrum of the system is then obtained by considering the combined effect of all noise sources. The model may be considered in several sections, as indicated by the boxes in Fig. 2. The boxes correspond to the domains within Fig. 1. We apply the following inputs to the flexure: (1) Force or displacement signal, $\Psi$, (2) mechanical environmental noise, e.g. vibrations, $\sigma_{MN}$, and (3) thermomechanical displacement noise, $\sigma_{Mn}$, which has a spectral density, $S_{Mn}(f)$.

$$S_{Mn}(f) = 4k_B T \frac{2\pi f}{\hbar \omega_h}$$  \hspace{1cm} (1)

In Eq. (1), $k_B$ is Boltzmann’s constant, $T$ is temperature in Kelvin, $k$ is the stiffness, $f$ is the noise frequency in Hz, and $\omega_h$ is the natural frequency of the compliant structure in radians per second [14]. Ambient mechanical and thermal noise spectral densities were measured in our laboratory.

The mechanical displacement noise sources are scaled by a constant, $\Lambda$, to match the units of the signal, where $\Lambda$ has either a unity value for displacement signals or value of $k$ for force signals. The compliant structure, i.e. flexure, acts as a non-dimensional mechanical filter, $F_F(s)$, to the signal as expressed by the Laplace transform of the dynamics of the compliant structure. All filters, $F(s)$ in the model are assumed to have unity steady state gain. The compliant structure also acts as a mechanical transducer that converts force and displacement into strain. This is represented in the model as the flexure gain, $\varepsilon_F$. The expressions for rectangular flexures with common types of boundary conditions are provided in Table 1. $L_h$, $b_h$, and $h_h$ are the length, width and height of the flexure. The variable, $E$ represents Young’s Modulus and $N_b$ is the number of flexures that mechanically act in parallel to form the compliant structure.

<table>
<thead>
<tr>
<th>TABLE 1: Common forms of the flexure gain, $\varepsilon_F$.</th>
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<tbody>
<tr>
<td>Fixed-Guided</td>
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<td>$\delta$</td>
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<td>$F$</td>
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The flexure transfer function exhibits a thermal sensitivity $\alpha_{FG}$, when $E$ is sensitive to changes in Wheatstone bridge temperature, $\sigma_{STC}$. The gain, $G_{SG}$, accounts for the averaging of the strain field over the finite geometry of the piezoresistor in the length and thickness dimensions.

$$G_{SG} = \left(1 - \frac{L_c + 2L_t}{\gamma L_c} \right) \left(1 - \frac{h_h}{h_t} \right)$$  \hspace{1cm} (2)

In Eq. (2), $L_c$ is the length and $h_t$ is the height of the piezoresistor. The sensor is placed a distance $L_c$ from the ground of the flexure. The constant $\gamma$, has value of 1 for fixed-guided or 2 for fixed-free flexures. The second term is normally neglected for surface mounted or deposited piezoresistors since $h_t$ is usually much smaller than $h_h$.

The signal is transformed from the mechanical to electrical domains via the Wheatstone bridge. The bridge sensitivity to strain depends upon the bridge strain type, $N_c$, which is defined as the number of strain sensitive resistors in the bridge. The gauge factor, $G_F$, exhibits thermal sensitivity $\alpha_{GF}$. The thermal sensitivity is defined by the number of thermally active resistors on the bridge, $N_{Th}$, and the temperature coefficient of
resistance, \( \sigma_{Rw} \). Likewise for other resistors forming the bridge but located elsewhere, \( N_{Tr} \) and \( \sigma_{Rv} \). The imbalance in the bridge resistors is represented by the dimensionless term \( \Delta_{Rw} \). The output signal is scaled by the bridge energizing voltage \( V_S \). The sensor noise, \( \sigma_{Vw} \), is composed of Johnson and flicker noise contribution from each of the 4 piezoresistors. This noise exhibits spectral density \( S_{Vw}(f) \).

\[
S_{Vw}(f) = 4k_{TR} + \frac{1}{16}\sum \alpha \frac{V_S^2}{C_\omega Q f}
\]

In Eq. (3), \( R \) is resistance, \( \alpha \) is the Hooge constant, \( C_\omega \) is the carrier concentration, and \( Q \) is the volume of the piezoresistor [7, 15]. The bridge voltage is attenuated by the gain of the STC, \( \alpha_{STC} \), which has thermal sensitivity, \( \alpha_{STC} \).

\[
G_{STC}(T) = \frac{R_w}{R_w + R_{STC}} \left[ 1 + \frac{R_{STC}}{R_w} (\alpha_{nw} - \alpha_{Rw}) T \right]
\]

The STC attenuates the bridge energizing voltage through the use of resistors, \( R_{STC} \), with high thermal sensitivity, \( \alpha_{RSTC} \), in series with the bridge resistance \( R_w \). These resistors are used to cancel the thermal variation in the signal due to \( \sigma_{GF} \) and \( \sigma_{TP} \).

\[
R_{STC} = \frac{R_w (\alpha_{Tr} + \alpha_{T})}{\alpha_{RSTC} - \alpha_{nw} - \alpha_{GF} - \alpha_{TP}}
\]

The STC and bridge resistors are separated by some distance; therefore they may experience different temperatures. The transfer function \( F_T(s) \) captures this by modeling the system as a thermal low pass filter. The thermal variations occur at relatively low frequencies, therefore the bandwidth of \( F_T(s) \) is large enough to approximate as unity.

The bridge energizing voltage has a thermal sensitivity, \( \alpha_{GF} \), to changes in its temperature, \( \alpha_{TP} \). Electrical noise from the energizing voltage, \( \sigma_{Vpr} \), and power supply noise as attenuated by the power supply rejection ratio \( PSRR(s) \) are added to the signal. A filter, \( F(s) \), is used to reduce the noise from the voltage source. The bias voltage is treated as equal to the bridge energizing voltage.

The instrumentation amplifier gain, \( G \), has a thermal sensitivity \( \alpha_{G} \) to the amplifier temperature, \( \alpha_{Ta} \). The gain is calculated by constraining the maximum input to the ADC to a fraction of its operating range, \( u \), which is generally 0.9, or 90% of the ADC’s voltage range, \( V_{range} \).

The maximum signal is found by substituting in the flexure’s yield stress, \( \sigma_y \) as modified by a safety factor, \( \eta \), that protects against failure of the flexure’s material. After the preceding we have:

\[
G = \frac{\nu V_{range} E \eta}{2 \sigma_y G_{SG} N_T G_{STC} V_S}
\]

The signal is biased by \( V_B \) at the output of the amplifier. The error terms associated to noise \( (\sigma_{Vai}, \sigma_{Vao}) \), offsets \( (\Delta_{Vai}, \Delta_{Vao}) \) and the thermal sensitivities of the offsets \( (\alpha_{Vai}, \alpha_{Vao}) \) are added to the signal. The variations in the common mode of the bridge output are attenuated by the common mode rejection ratio, \( CMRR(s) \). The same is done for power supply variations through the power supply rejection ratio, \( PSRR_A(s) \).

The variation in the supply voltage, \( V_p \), is caused by thermal sensitivity, \( \sigma_{Vp} \), to the supply temperature, \( \sigma_{Tp} \), electrical noise \( \sigma_{Vp} \) and ripple at the input to the supply circuit, \( \sigma_{Vr} \). The ripple is attenuated by the ripple rejection ratio \( RRR \). A filter, \( F_R(s) \), should be used to reduce the noise from the power supply.

The output voltage signal is read in from the amplifier to an ADC. The ADC adds electrical noise, \( \sigma_{VC} \), thermal sensitivity, \( \sigma_{VT} \), to the ADC temperature, \( \sigma_{Ta} \), and an offset \( \Delta_{VC} \) to the signal.

The digital signal is passed through a digital filter, \( F_D(s) \), which may be adjusted to attenuate noise outside of the signal spectrum. The signal is scaled by a calibration coefficient, \( C \), which is found by enforcing equality between \( \Psi_M \) and \( \Psi \).

\[
C = \frac{1}{e_F G_{SG} N_T G_{G} G_{STC} V_S G}
\]

When multiple sensors are used to obtain multi-axis measurements, a coordinate transform matrix, \( A \), is used to transform the vector of sensor readings, \( p \), into a vector of the values in the axes of interest, \( v = Ap \). Uncorrelated noise from each sensor is attenuated by the averaging effect of combining the multiple sensor readings, captured by \( M \), which may be written as a vector to calculate the performance of the jth axes of interest.

\[
M_j = \sqrt{\sum_k A_{j,k}^2}
\]
**Dominant Noise Sources and System Characteristics**

Partial derivatives for the dominant noise sources, $\sigma_{v_{in}}$, $\sigma_{v_{bias}}$, $\sigma_{T_{env}}$ are listed below.

\[
\frac{\partial \Psi_{V_{m}}(s)}{\partial \sigma_{v_{in}}} = MCF_{c}(s)G \\
\frac{\partial \Psi_{V_{m}}(s)}{\partial \sigma_{v_{bias}}} = MCF_{c}(s) \\
\frac{\partial \Psi_{V_{m}}(s)}{\partial \sigma_{T_{env}}} = MCF_{c}(s)
\]

The system is evaluated using spectral analysis where all noise sources are considered as unbiased, uncorrelated, and normally distributed with spectral densities, $S_{n}(f)$. The spectral densities are scaled by their respective frequency dependent sensitivities and geometrically summed to obtain the system noise spectral density, $S_{\psi_{m}}(f)$.

\[
S_{\psi_{m}}(f) = \sum_{n} \frac{\partial \Psi_{V_{m}}(2\pi if)}{\partial \sigma_{n}} \left(\frac{\partial \Psi_{V_{m}}(2\pi if)}{\partial \sigma_{n}}\right)^{*} S_{n}(f)
\]

**INSIGHTS FROM THE MODEL**

**Electronic Sources**

The model is generalized so that it may be used with a wide range of applications. Through this model, we may gain insight on best design of general and specific sensor systems. The model assumes the use of high-performance electrical components – instrumentation amplifier (Analog Devices AD624), voltage source and bias (Texas Instruments REF50xx series), and ADC (National Instruments 9215 ADC). This is essentially a best practice that ensures that these electronics are not a significant source of noise. There relevant noise values for these components are provided in the references. We will shortly show that sensor noise is the dominant noise source in well-designed sensing systems; therefore AC bridges are typically not needed to reduce amplifier noise.

When the resistance of the piezoresistor is low, noise from the instrumentation amplifier may become significant. The input noise from the amplifier is added to the signal before the signal is amplified, therefore reducing the signal output by the bridge may make the amplifier input noise the dominant source. If the signal is small, thermomechanical noise, i.e. random vibration in the beam caused by energy exchange between the flexure and environment, may become a major noise source once propagated via the electronics.

**Mechanical Sources**

The mechanical noise sources do not significantly contribute to the overall noise in most sensor systems because this is attenuated by physical filters (e.g. via optical tables) before they reach the measurement signal.

**Thermal Sources**

Errors caused by thermal fluctuations can generally be avoided by proper system design. The Wheatstone bridge may be thermally balanced by placing the bridge resistors close together so that they are subject to the same temperature. Similarly, STC resistors may be used to make the gauge factor and flexure transfer function effectively thermally insensitive. Bridge offsets generated by manufacturing inaccuracies are compensated for by the bias voltage. The thermal sensitivity of the bridge offset, however, is unaffected by the bias voltage as may be discerned from Eq. (9). In cases where the manufacturing inaccuracies are large, it may become necessary to minimize thermal fluctuations in the environment through the use of insulation or active temperature controls. In most cases, this type of thermal control is not necessary since relative manufacturing inaccuracies are typically small in MEMS. In most cases, noise in the piezoresistor itself limits the resolution of the sensor system.

**Johnson and Flicker Noise**

The noise in the sensor may be broken into two sources: (1) Johnson noise caused by the thermal agitation of electrons in a conductor and (2) Flicker noise caused by conductance fluctuations that manifest during the capture and release of charge carriers in the piezoresistor [15]. Doping concentration affects resistivity, gauge factor and carrier concentration of silicon piezoresistors, therefore silicon piezoresistors may be Johnson or flicker noise dominated. There is a tradeoff between noise and sensitivity as dopant concentration is varied.

In the case where the performance of the sensor is limited by flicker noise, an optimal sensor length and thickness will exist. As the length and thickness of the sensor increases, the sensor volume and therefore number of carriers increases. This acts to decrease flicker noise. The average amount of strain in the sensor also
decreases as the length and thickness of the sensor increase. In balancing these two effects, optimal length and thickness may be found. The optimal sensor length is \( \gamma/3 \). The fixed-guided condition and other boundary conditions are often found in multi-axis flexures. The optimal sensor to flexure thickness ratio is 1/3, which is consistent with prior force sensor work \[7\].

**PIEZORESISTIVE SENSOR DESIGN**

**Reduced Piezoresistive Sensor System Model**

One of the most important parameters describing sensor performance is the dynamic range, i.e. the ratio of range to resolution of the system. The range and resolution are functions of the flexure geometry but the dynamic range is typically dependent only on the piezoresistor itself. Therefore, it is generally best to optimize the sensor system to achieve the highest practical dynamic range. From the model it was determined that the three largest noise sources were the Johnson noise, flicker noise and instrumentation amplifier noise. In the simplified model, only these three noise sources are passed through the system model to create a reduced expression for the resolution of the sensor. The dynamic range, \( DR \), of the sensor is given in Eq. (11),

\[
DR = \frac{\sigma_r N_i G_{stc} V_s \left( 1 - \frac{L_r + 2L}{\gamma L_y} \right) \left( 1 - \frac{h_r}{h_y} \right)}{\eta EM \sqrt{4k_{TRB} + \frac{1}{16} \sum_{i=1}^{4} \alpha_r V_s^2 \ln(r) + S_{val}} \cdot R}.
\]

where \( R = \frac{\rho N_i^2 L_r}{b_r h_y} \), \( \eta = L_r b_r h_y \),

\[
S_{val} = \frac{1}{16} \sum_{i=1}^{4} \alpha_r V_s^2 \ln(r)
\]

The bandwidth of the sensor increase. In balancing these two effects, optimal length and thickness may be found. The bandwidth of the sensor may be written as a function of the signal frequency where the pole of the software first order, low pass filter is placed at a multiple \( r \) of the signal frequency, \( f_{\text{sig}} \). The approximation of this bandwidth is given by Eq. (12) \[15\].

\[
B = \frac{\pi}{2} (r - 1) f_{\text{sig}}
\]

This simplified model makes it possible to optimize the dynamic range of the sensing system for most cases. However, when very small forces or displacements are being measured, the thermomechanical noise may become greater than the noise from the instrumentation amplifier.

**EARLY RESULTS AND DISCUSSION**

The noise characteristics of a simple quarter bridge \( (N_r=0.25) \) polysilicon piezoresistive sensor was compared to model predictions as shown in Fig. 5. The sensor and electronics are shielded from external noise sources. The sensor is located on a large aluminum thermal reservoir within a Faraday cage. The flicker noise characteristics of the polysilicon piezoresistive sensor were experimentally determined. The spectral density of the noise was measured from 0.01 Hz to 5 kHz, corresponding roughly to the common range of operation for such sensors.

![FIGURE 5: Polysilicon Piezoresistive sensor noise spectrum compared to predictions.](image)

The model indicates that the sensor flicker noise should be the dominant source over the full range of measurement when the bridge is energized at 10 V. This prediction is verified by the measured spectral density. The predicted and measured noises are 77 mV and 78 mV respectively. The model also correctly predicts the change in noise spectral density resulting from a reduction in the bridge energizing voltage from 10 V to 3 V. In the reduced voltage scenario, the predicted and measured noises are 23 mV and 21 mV respectively.

In the third scenario studied in the experiment, the electrical and thermal shielding surrounding the polysilicon piezoresistor was removed to deliberately exposed the sensor to random temperature variations (‘Exp. Data’). The spectral density of these temperature variations was measured and propagated through the system model to predict the effect of exposing the sensor on the noise spectral density. The electrical noise prediction was unaffected by this change, however the thermal noise component of the prediction rose significantly to become a
dominant source over the low frequencies (0.01 to 1 Hz) as shown in Fig 6. This effect was observed in the measurements of the spectral densities with and without thermal shielding. This indicates that thermal effects on system noise can effectively be integrated into a cohesive model as described in the previous sections.

\[ \begin{array}{c}
10^{-2} & 10^{-1} & 10^{0} & 10^{1} \\
10^{-5} & 10^{-4} & 10^{-3} & 10^{-2} & 10^{-1} & 10^{0} \\
\end{array} \]

**FIGURE 6: Measurement of noise spectral densities with and without thermal shielding.**

REFERENCES


