INTRODUCTION
Angular encoder accuracy is often the key limiting factor in rotational metrology systems. Inconsistent spacing of the graduations and spindle error motion are major sources of encoder error. Through calibration, the repeatable components of encoder error can be eliminated. Calibration systems have been developed to measure the error map at 128 angular intervals, achieving 0.01" (arc-sec) measurement uncertainty at [1]. Another method using five evenly distributed scanning-heads, self-calibrated graduation errors except 5th order harmonic multiples [2].

This paper presents an encoder self-calibration algorithm for precision rotary stages following original research in [3]. This calibration method can be performed over a large speed range with the encoder on its final axis, and can capture error associated with each position count. Early calibration was performed on an air-bearing spindle with an integrated rotary encoder that was not precise enough to investigate limitations of the algorithm. A precision rotary encoder setup has been designed for this investigation, with a goal of 0.001" measurement uncertainty. To achieve this, the algorithm has been improved to be less sensitive to timing resolution and rotary vibrations.

CALIBRATION THEORY
Most incremental encoders work on the same basic principle. Physical markings on the encoder scale are read by the scanning unit and output as a pair of quadrature signals (A/B) and an index signal indicating the start of each revolution. The rising/falling edges of these signals are treated as spatial sampling events that mark spindle rotary position (FIGURE 1).

FIGURE 1. Encoder spatial sampling events [3]

Deviations in actual graduation spacing from nominal spacing affect encoder accuracy. Encoder calibration aims to identify these deviations for measurement compensation. On the basis of earlier work [3], an improved algorithm is developed, as summarized below.

The time at which each spatial sampling event occurs is measured using custom high-speed processing electronics. A dynamic model of the system is used to relate spatial widths to time measurements. The spindle dynamics are modeled as an inertia and damping element, shown below as a differential equation.

\[
d\omega/dt + \omega = 0
\]  \hspace{1cm} (1)

The normalized damping coefficient (c, ratio of spindle damping to rotor inertia) is modeled to include constant and speed dependent damping,

\[
c = c_0 + c_1(\omega - \omega_0)
\]  \hspace{1cm} (2)

where, \(\omega_0\), is the initial spindle speed. The angular speed at each sampling event is found by solving a simplified version of the differential equation, using a 2nd-order Taylor series expansion to give:

\[
\omega_k = \omega_0 + ak + bk^2
\]  \hspace{1cm} (3)

where \(a = -c_0\Delta_0\) and \(b = c_0c_1(\Delta_0)^2/2\) are damping coefficients estimated below. From the time measurements spindle speed is estimated.

\[
\omega_k = \Delta_k/T_k
\]  \hspace{1cm} (4)

This equation cannot directly be used for encoder calibration while initial spindle speed and damping are undetermined. Circular closure is used to solve for the initial spindle speed. Dynamic reversal is used to yield a linear equation to solve for damping coefficients:

\[
[U_1, V_1, -U_2, -V_2] \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{bmatrix} = [m_1 - m_2]
\]  \hspace{1cm} (6)

where \(U_1, V_1, U_2, V_2\), \(m_1\) and \(m_2\) are vectors containing the time measurement results. A least-square fitting is used to estimate the damping coefficients.

Using (6) damping estimates are overly sensitive to uncertainties caused by limited timing resolution. The proposed method eliminates this sensitivity by using spatial integration to increase the dynamic range of the measurement patterns with respect to time. Instead of equation (6) the modified linear equation becomes,

\[
\text{int}([U_1, V_1, -U_2, -V_2]) \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{bmatrix} = \text{int}(m_1 - m_2)
\]  \hspace{1cm} (7)

where the \(\text{int}(\cdot)\) operator represents a cumulative matrix row summation. A least squares fit is
applied and the damping parameters are estimated as:
\[
\begin{bmatrix}
\bar{a}_1 \\
\bar{b}_1 \\
\bar{a}_2 \\
\bar{b}_2
\end{bmatrix} = (\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T[\text{int}(m_1 - m_2)]
\] (8)

The spatial widths \( \bar{\Delta} \) of each encoder count are derived and the encoder error map \( \bar{H}(k) \) is found, representing a combination of grating error and radial error motion.
\[
\bar{\Delta} = m_1 + a_1u_1 + b_1v_1
\] (9)
\[
\bar{H}(k) = \sum_{i=1}^{N} (\bar{\Delta}_i - \Delta_0) - \sum_{i=1}^{N} \sum_{i=1}^{N} \frac{\Delta_1 - \Delta_0}{N}
\] (10)

**SIMULATION RESULTS**

Simulation is used to evaluate the theoretical accuracy limit of the calibration algorithm. Precise time measurements are generated at each encoder count using the simulated spindle dynamics from (1), experimental damping coefficients and a reference error map. The time measurements are quantized to a timing resolution of 1.2GHz, to simulate the high speed processing electronics in the experimental setup. The simulated time measurements are similar to those generated from the experimental setup but only reflect timing uncertainty. Self-calibration is performed on the simulated time measurements and a new calibrated error map is found. The accuracy of the method is found as the rms difference between the reference and calibrated error maps. The simulated accuracy of self-calibration is expected to follow time quantization error, and increases with the speed of rotation. Accuracy of self-calibration with and without spatial integration is compared for a speed range from 10 to 1000rpm (FIGURE 2). Spatial integration is necessary for accurate system identification. This ensures the uncertainty introduced due to limited timing resolution is minimized and the theoretical limit is achieved.

![FIGURE 2: Simulated self-calibration accuracy.](image)

**EXPERIMENTAL SETUP**

A precision rotary table, high-speed electronics and software have been developed to experimentally investigate the limits of the self-calibration algorithm [4]. The precision rotary table consists of an air-bearing spindle (Professional Instruments 10R block-head), and a rotary encoder (Heidenhain 4282C), shown in FIGURE 3. The rotary encoder has 32768 graduation lines, 131072 counts/rev and specified system accuracy of \( \pm 2'' \).

![FIGURE 3: Precision encoder experimental setup.](image)

**SINGLE HEAD SELF-CALIBRATION**

Self-calibration was first investigated using a single encoder scanning head in position H1. FIGURE 5 shows the encoder error map obtained at 270rpm with 500 harmonic components included. The error map is dominated by a 1cpr component, largely influenced by graduation scale eccentricity, which is only captured with on-axis calibration methods. The experimental repeatability of calibration is determined as the rms difference between two error maps derived sequentially. Repeatability measurements show the same pattern as the
simulated results, which are proportional to speed due to increasing timing uncertainty (FIGURE 6). Experimental repeatability measurements do match the predicted simulation accuracy due to limited signal to noise ratio on the encoder signal causing additional uncertainty.

FIGURE 5: Single head calibration at 270rpm. Encoder error with 500cpr harmonics included (top), error map harmonic amplitude (bottom).

FIGURE 6: Calibration repeatability

FIGURE 7 shows the first 6 harmonic components of error maps across speed, by continuous time measurement and calibration. The amplitudes of error map harmonics change significantly with speed. This was identified as rotary vibration caused by disturbance torque during the spindle free response. This vibration, not modeled in the self-calibration algorithm, can introduce error into the calibration. As a result, the calibrated error map from (10) for encoder head H1 contains not only grating error $G(\theta)$ and radial error motion $Y(\theta)$ components, but also rotary vibration $V(\theta)$. The error map can be modeled as

$$H1(\theta) = G(\theta) + Y(\theta) + V(\theta)$$

(11)

The rotary vibration component can be compensated using a least-square fitting on individual error map harmonics assuming grating error and radial error motion are constant and rotary vibration is proportional to the inverse of speed squared.

The grating error component rotates with head position, error motion along X or Y will only effect measurement when orthogonal to the head and rotary vibration is rotor position dependent and the only component identical among heads. FIGURE 8 shows error maps from four heads calibrated at 100 rpm.

FIGURE 7: Results of head H1 error map harmonics (grating error, radial error motion and rotary vibration). Measured (solid line) and vibration compensated (dashed)

MULTIPLE HEAD COMPARISON

To more accurately remove rotary vibration from the calibrated error map, a multiple scanning head comparison method was developed. Each calibrated error map from individual scanning heads is composed of a unique combination of grating error, radial error motion along X or Y and rotary vibration.

$$
\begin{align*}
H1(\theta) &= G(\theta) + Y(\theta) + V(\theta) \\
H2(\theta) &= G(\theta - \pi/2) - X(\theta) + V(\theta) \\
H3(\theta) &= G(\theta - \pi) - Y(\theta) + V(\theta) \\
H4(\theta) &= G(\theta - 3\pi/2) + X(\theta) + V(\theta)
\end{align*}
$$

(12)

The grating error component rotates with head position, error motion along X or Y will only effect measurement when orthogonal to the head and rotary vibration is rotor position dependent and the only component identical among heads. FIGURE 8 shows error maps from four heads calibrated at 100 rpm.

FIGURE 8: Individual scanning head error maps at 100rpm

Each component of the calibrated error map can be isolated using combinations of measurements from the four heads. To remove rotary vibration, error maps from each scanning head are summed,
cancelling radial error motion of opposing heads and grating error harmonics. The extracted rotary vibration component at 100rpm is shown in FIGURE 9. FIGURE 10 shows the rotary vibration over the speed range where a -40dB/decade slope is observed. This disturbance torque doesn’t change with spindle speed, but depends only on the rotary position.

FIGURE 9: Extracted rotary vibration at 100rpm.

FIGURE 10: Extracted rotary vibration over 80 to 270rpm speed range.

The multiple head comparison method allows rotary vibration to be identified and eliminated from the measurement. FIGURE 11 shows the compensated error map harmonics of head H1 by removing the spindle rotary vibration components, constant over a large speed range. Results from this analysis match estimation performed on single head error maps harmonics in FIGURE 7.

Compensated error maps for 100rpm are shown in the top graph of Figure 12. The error map variation from 100 to 200rpm is less than 0.01“ rms (bottom graph of FIGURE 12). For better precision, four encoder heads can be averaged to get more accurate spindle angle measurement. The uncertainty of averaged measurements between 100 rpm and 200 rpm is less than 0.005” (k=2).

FIGURE 12: Compensated error map of all four heads.

CONCLUSION
A novel self-calibration algorithm for precision rotary encoders has been developed that uses a spatial integration technique to improve the accuracy of inertia and damping coefficient estimations. A precision rotary table setup was used to investigate the performance limits of the algorithm. A rotary vibration caused by disturbance torque, was identified from encoder calibration results. A method using multiple scanning heads was developed to extract the rotary vibration from the error map and eliminate it. Compensated error maps are repeatable within 0.01” for single head and 0.005” (k=2) for four head averaging, compared at spindle speeds of 100 and 200rpm.

ACKNOWLEDGEMENT
This research is sponsored by KLA-Tencor, NSERC, and Canada Foundation for Innovation.

REFERENCES