INTRODUCTION
Miniaturized machine tools have been commonly used for mechanical micro machining of meso/micro scale components. Precise rotation of spindle is essential for accurate positioning of micro cutting tools or work pieces. Measurement and analysis of spindle error motions provides useful insights into the machining accuracy of machine tool [1]. Conventional spindle error measurement methods require multiple capacitive sensors and instrumentations and it is difficult to be implemented in testing the rotational accuracy of miniaturized machine tool spindle due to space limitations. In the present work, single point asynchronous motion (SPAM) test is carried out for evaluating spindle radial accuracy of a miniaturized machine tool using a capacitive sensor and cylindrical artifact in accordance with ANSI/ASME B.89.3.4.M [2]. Figure 1 shows the experimental arrangement of the SPAM test for the miniaturized machine tool spindle.

Measurement data shows a predominant sinusoidal pattern and super imposed harmonic components. It includes the contribution of centering error, form error of artifact and spindle radial error motion. Evaluation of spindle radial error motion requires accurate estimation of harmonic components of measurement data. Prominently used Fourier transform method is not suitable for harmonic analysis of time sampled measurement data [3].

To overcome this difficulty, in the present work, a least square method is used for the harmonic analysis of measurement data. Sum of sinusoidal functions are used for characterizing harmonic components of measurement data.

FIGURE 1. Measurement setup for SPAM test.

Surface of the artifact is used as a reference for measuring radial error motions of the spindle. Measurement data is acquired using a computer aided data acquisition system in discrete time intervals. Sampling frequency of spindle radial error measurement \( f_s \) is chosen in such a way that it is at least twice the product of desired cycles per revolution \( H \) and spindle rotational frequency to avoid the aliasing effects [2]. Miniaturized machine tool spindle exhibits speed variations during spindle radial error measurement. Figure 2 shows the measurement data obtained from the capacitive sensor for the spindle speed of 50,000 rpm.

FIGURE 2. Measurement data obtained at the spindle speed of 50,000 rpm.
Experimental results of the proposed method are demonstrated in comparison with Fourier transform method.

**LEAST SQUARE METHOD FOR HARMONIC ESTIMATION OF MEASUREMENT DATA**

This work extends the best fit sine wave method proposed by Murthy [4], for analyzing the harmonic components of measurement data using a mathematical model consisting of a sum of sinusoidal functions as given by equation (1).

\[
m_i = C + \sum_{h=1}^{H} a_h \cos(2\pi hf_0 t_i) + b_h \sin(2\pi hf_0 t_i) \quad i = 1, 2, 3, \ldots, n
\]  

(1)

In equation (1), \(a_h, b_h\) represent amplitudes of harmonic components of measurement data. \(C\) corresponds to the average position of artifact. \(f_0\) is the fundamental frequency of measurement data. \(H\) represents specified harmonic cutoff in cycle per revolution.

If \(m'_i = [m'_{i1}, m'_{i2}, \ldots, m'_{in}]^T\) is a set of \(n\) samples of measurement data obtained in the SPAM test, harmonic components of measurement data is assumed to follow the equation (1) and it is represented in matrix form as shown in equation (2).

\[
m'_i = Dx
\]  

(2)

Where \(D\) is the matrix containing \(n\) rows and \(2H+1\) columns as given by equation (3).

\[
D = \begin{pmatrix}
\cos(2\pi ft_1) & \sin(2\pi ft_1) & \ldots & \cos(2\pi Hf_0 t_1) & \sin(2\pi Hf_0 t_1) \\
\cos(2\pi ft_2) & \sin(2\pi ft_2) & \ldots & \cos(2\pi Hf_0 t_2) & \sin(2\pi Hf_0 t_2) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\cos(2\pi ft_n) & \sin(2\pi ft_n) & \ldots & \cos(2\pi Hf_0 t_n) & \sin(2\pi Hf_0 t_n)
\end{pmatrix}
\]  

(3)

\(x\) is the set of model parameters as given by (4).

\[
x = [C, a_1, b_1, a_2, b_2, \ldots, a_H, b_H]^T
\]  

(4)

Following assumptions are made for estimating the model parameters using least square method.

- Model for estimating the harmonic components of measurement data is true
- Residual errors follow gaussian distribution with zero mean and variance.

Model parameters are estimated by minimizing the sum of squares of residuals (ssr) as given by equation (5).

\[
ssr = \sum_{i=1}^{n} e_i^2 
\]  

(5)

Here \(e_i\) represents the residual error of the fitted model to the measurement data as shown by equation (6).

\[
e_i = m_i - \hat{C} - \left(\sum_{h=1}^{H} \hat{a}_h \cos(2\pi hf_0 t_i) + \hat{b}_h \sin(2\pi hf_0 t_i)\right)
\]  

(6)

Least square solution vector of model parameters for the given value of fundamental frequency \(f_0\) is obtained using equation (7).

\[
\hat{x} = \left[(D^T D)^{-1} D^T\right]m'_i = [\hat{C}, \hat{a}_1, \hat{b}_1, \ldots, \hat{a}_H, \hat{b}_H]
\]  

(7)

Amplitude and phase of harmonic components can be obtained from equations (8) and (9).

\[
\hat{C}_h = \sqrt{\hat{a}_h^2 + \hat{b}_h^2}
\]  

\[
\hat{\gamma}_h = \tan^{-1}\left[-\frac{\hat{b}_h}{\hat{a}_h}\right]
\]  

(8)

(9)

It is noted that equation (7) requires the estimation of fundamental frequency of measurement data and it is obtained using following iterative method.

**Estimating of fundamental frequency measurement data**

Sum of squares of residuals given by equation (5) is written in matrix form using (2) and (3).

\[
ssr = (m'_i - D\hat{x})^T (m'_i - D\hat{x})
\]  

(10)

This criterion can be expressed with respect to \(\hat{x}\) and \(f_0\) can be determined using one dimensional grid search method for the maximum of \(g(f_0)\) [5, 6].

\[
g(f_0) = m'_i^T D\left[(D^T D)^{-1}\right]D^T m'_i
\]  

(11)

Equation (11) is estimated for the discrete frequency values \(f_j, j = 1, 2, \ldots, M\) in specified interval \([f_{\text{min}}, f_{\text{max}}]\) around the spindle rotational frequency \(f_0\) with a resolution \(\Delta f\). Grid search iteration method is used to determine fundamental frequency of measurement data that maximizes the criterion given by (11).

**EXPERIMENTAL RESULTS**

Proposed least square method is applied to the measurement data obtained at various spindle speeds. Harmonic estimation results of proposed method is compared with Fourier transform method. Experimental results are demonstrated for the measurement data obtained at the spindle speed of 50,000 rpm.
Estimation of fundamental frequency

A frequency interval of 827-834 Hz is chosen for determining the fundamental frequency of measurement data, as the spindle rotational frequency \( f_r \) is 833.33 Hz. Values for the criterion given by equation (11) are obtained at the frequency resolution of 0.01 Hz. Fundamental frequency of measurement data is estimated as 830.69 Hz using grid search method for the maximum value of criterion and the results are shown in Figure 3(a).

Estimation of harmonic components of measurement data

Experimental results of least square method for the first 10 harmonic components of measurement data is listed in Table.1 and it is compared with results obtained from Fourier transform method.

Table 1. Harmonic estimation results for measurement data obtained at 50,000 rpm

<table>
<thead>
<tr>
<th>Number of harmonics</th>
<th>Amplitude of harmonics (µm) Least square</th>
<th>FFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.82845036</td>
<td>6.77224408</td>
</tr>
<tr>
<td>2</td>
<td>0.15301395</td>
<td>0.18002778</td>
</tr>
<tr>
<td>3</td>
<td>0.61388373</td>
<td>0.59221204</td>
</tr>
<tr>
<td>4</td>
<td>0.07481281</td>
<td>0.07817169</td>
</tr>
<tr>
<td>5</td>
<td>0.43201515</td>
<td>0.33542394</td>
</tr>
<tr>
<td>6</td>
<td>0.0569406</td>
<td>0.04648886</td>
</tr>
<tr>
<td>7</td>
<td>0.18841397</td>
<td>0.11205739</td>
</tr>
<tr>
<td>8</td>
<td>0.03663476</td>
<td>0.01845957</td>
</tr>
<tr>
<td>9</td>
<td>0.09647231</td>
<td>0.02752655</td>
</tr>
<tr>
<td>10</td>
<td>0.04171377</td>
<td>0.02889850</td>
</tr>
</tbody>
</table>

From the results shown in Table. 1, it is observed that Fourier transform method gives smaller values for the amplitude of most of the harmonic components as compared to the least square method, due to the spectral leakage effect. It is also noted that the amplitude of first harmonic or fundamental component of measurement data is dominant among the other harmonic components of measurement data.

Figure 3 (b) shows the frequency content of measurement data obtained from Fourier transform method. It exhibits spectral leakage to the adjacent frequency bins due to spindle speed fluctuations.

Figure 4 shows the improved estimation results of least square method for the fundamental harmonic component for various spindle speeds.
harmonic component of measurement data obtained at various spindle speeds.

It can be seen that magnitude of fundamental component increases with increase in spindle speed. It is due to the centering error and unbalance mass of the artifact.

**Performance of least square fitting method**

Performance of proposed method was tested for the short duration measurement data with incomplete number of spindle revolutions and the experimental results are given in Figure 5. It can be seen from Figure 5 (a) that proposed model closely follows the periodic trend of the measurement data. Correlation coefficient ($R^2$) of the proposed model is found as 0.989.

![Graph of least square fitting](image1)

(a) Least square fitting of proposed model to the measurement data.

![Graph of residual errors](image2)

(b) Residual errors of measurement data

**FIGURE 5. Results of least square method for the measurement data obtained at 50,000 rpm.**

Residual errors of measurement data are centered on the mean line without any trend pattern as shown in Figure 5 (b), satisfying the assumptions of least square method. These results prove the suitability of proposed model and least square method for analyzing the harmonic components measurement data.

**CONCLUSIONS**

Proposed least square method is found to be suitable for the harmonic analysis of spindle radial error data obtained in discrete time interval. Fourier transform method does not provide accurate values for the harmonic components of measurement data due to spectral leakage. The proposed method is suitable for analyzing short duration measurement data with incomplete number of spindle revolutions. The present algorithm requires no frequency domain conversion for the harmonic analysis of measurement data, consequently it can be used for online measurement and analysis of spindle error motions of miniaturized machine tool.

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**REFERENCES**


