THE DESIGN AND OPTIMIZATION OF CASCADED CHEVRON FLEXURES AND ACTUATORS FOR PRECISION MOTION GUIDANCE

Shih-Chi Chen\textsuperscript{1,2} and Martin L. Culpepper\textsuperscript{2}
\textsuperscript{1}Wellman Center for Photomedicine
Massachusetts General Hospital, Harvard Medical School
Boston, MA
\textsuperscript{2}Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA

ABSTRACT
This paper presents modeling and optimization procedures for optimizing the stiffness and transmission ratio of cascaded chevron flexures that are used as actuators and/or transmissions in small-scale precision machines. The principle of mechanical impedance matching is used to optimize the mechanical performance of the chevrons. This opens the way to optimization of electrical and thermal characteristics of chevron actuators. The utility of this approach is demonstrated in the context of a microactuator for a micro-optical scanning system.

INTRODUCTION
Chevron flexures are used as transmissions \cite{1-4} that either (i) transform the output of high-force, low-stroke actuators (e.g. piezoelectric actuators) to a different force-stroke combination or (ii) serve as the transducing and transmission of strain-based actuators (e.g. thermomechanical actuators). A conventional chevron flexure is shown at the top of Figure 1A, and several design parameters are shown in Figure 1B.

Chevrons are commonly used with piezoelectric and \(\mu\)-thermomechanical actuators (\(\mu\)-TMAs). In the former case, the piezo actuator drives the input of the chevron, \(x_i\), to achieve larger output stroke, \(x_o\). In the later, \(x_i\) is fixed and a current is passed through the beam so as to yield an output displacement.

The transmission ratio \(T_r = x_o/x_i\), axial stiffness \((K_A)\) and lateral stiffness \((K_L)\) of a chevron flexure are a function of the chevron’s side angle, \(\theta\). As \(\theta\) increases from zero (i) the \(T_r\) and the lateral stiffness decrease sharply and (ii) the axial stiffness increases gradually. In many applications, one desires to place multiple chevrons that are cascaded, \textit{i.e.} in series, with the first chevron driving the input of a second chevron and so on. This results in sequential amplification of the input motion at the expense of stiffness. In this paper we provide a parametric model that may be used to manage the tradeoff between \(T_r\) and stiffness for chevrons in series. The demonstration of the modeling and its application are presented in the context of designing a \(\mu\)-optical scanner.

MICRO SCANNER LAYOUT
Figure 2 shows the scanner layout. The scanner is designed to route a laser and scan its focal point across a plane by translating a graded index (GRIN) lens and rotating a prism. The GRIN lens and prism actuator should possess a stroke of 100 \(\mu\)m and 2\(^\circ\) respectively. Figures 2A and 2B show that sets of chevron TMAs 1-2 and 4-5 serve as the transducing elements. The output of TMA sets 1-2 are amplified by chevron flexure 3. The output of TMA sets 4-5 are amplified via set 6 chevron flexures. The TMAs are contoured as shown at the bottom of Figure 1A to enhance their thermal and mechanical performance \cite{1}.
FIGURE 2: μ-scanner layout showing banks of parallel chevron actuators 1-2-4-5 and amplifying chevrons 3-6 (A) and actuator displacements that are amplified by the chevrons and resulting motion of the GRIN lens (G) and prism (P) (B).

MODELING AND DESIGN

The optimization of energy transfer through cascaded compliant machine elements may be achieved by matching their mechanical impedance, i.e. designing the chevron flexures so that the axial stiffness of the first chevron ($K_{A1}$) and lateral stiffness of the second chevron ($K_{L2}$) are equal. There are two ways to achieve equivalent stiffness values: (1) adjust $\theta_1$ and $\theta_2$ that are defined in Figure 3. This approach is best suited for applications wherein the chevrons are used as transmissions. (2) place $N$ of the first chevrons in parallel so that their combined axial stiffness equals the lateral stiffness of the second chevron. The dotted lines in Figure 3 indicate where additional chevrons could be added. This approach is best used when the first chevron serves as a transmission and actuator. For example a TMA.

FIGURE 3: A two–spring model for the cascaded chevron mechanism.

The relationship between the side angles and number of chevrons is plotted in Figure 4.

FIGURE 4: Transmission ratio, axial, lateral stiffness of a chevron mechanism as a function of angle. The axial stiffness may be increased by adding multiple chevron TMAs in parallel.

Cascaded chevron mechanism may be modeled as a two–spring system in Figure 3. The variable $X_{IN}$ represents the displacement of a chevron TMA (first chevron mechanism), and $X$ is the output displacement of the cascaded system. The relationship between $X$ and $X_{IN}$ is shown in Equation (1). The overall $T_r$ of the cascaded mechanism is shown in Equation (2). The stroke of cascaded system is the product of the input displacement and the system’s $T_r$, as shown in Equation (3).

$$X = \frac{1}{1 + K_{A1}(\theta_1)/K_{L2}(\theta_1)} \cdot X_{IN} \quad (1)$$

$$T_{TRANSMISSION} = T_{R1}(\theta_1) \cdot \left[ \frac{1}{1 + K_{A1}(\theta_1)/K_{L2}(\theta_1)} \right] \cdot T_{R2}(\theta_2) \quad (2)$$

$$S_{STROKE} = T_{R1}(\theta_1) \cdot \left[ \frac{1}{1 + K_{A1}(\theta_1)/K_{L2}(\theta_1)} \right] \cdot T_{R2}(\theta_2) \cdot X_{IN} \quad (3)$$

These equations replace intuition and iterative FEA as the common means of optimizing performance. This has been shown to yield a factor of 10X improvement [2-4] in the performance of cascaded chevrons.

Equation (2) was used to set a $T_r$ as a function of $\theta_1$ and $\theta_2$. A plot of the optimal $T_r$ for a given number of the TMAs was also obtained and the results are plotted in Figure 5.
Step 4: Obtain the corresponding values for $\theta_1$ and $\theta_2$ for the optimal $T_r$ from the transmission equation.

Based on the preceding approach, the following demonstrates the procedures that were used to engineer the GRIN lens actuators: (1) lowest temperature, and (2) minimum driving power. The design processes for the prism actuator is identical and will not be described.

**Design for temperature minimization**

The design process is: (1) determine the TMA’s maximum operating temperature from functional requirements, e.g., 150°C, (2) optimize the TMA’s performance, e.g. stroke, at the specified temperature by the geometric contouring method [1], and (3) obtain the required number of TMAs and the relative optimal $T_r$ from the surface plots that are shown in Figure 6. The surface plots were generated using Equation (2). The results are shown in Figure 6 and indicate that the optimal $T_r$ increases as (i) the number of TMAs increases and (ii) the values of $\theta_1$ and $\theta_2$ decrease. Accordingly, the required stroke and temperature may be achieved by selecting $N = 20$ TMAs and $\theta_1 = 2.0^\circ$ and $\theta_2 = 2.0^\circ$, where the optimal $T_r$ is 45.

**FIGURE 6: GRIN actuator performance**
Design for power minimization

As TMAs operate more efficiently at high temperature, the required displacement is achieved when a large portion of a TMA’s material operates at the highest safe operating temperature for silicon, e.g. 1200°K.

This example uses only two parallel TMAs to achieve a 100 µm output displacement with 125 mW power consumption, where the optimal $T_r$ is 28, for $\theta_1 = 2.5^\circ$, and $\theta_2 = 3.6^\circ$. Note $N = 2$ was selected so the required stroke may be achieved without exceeding 1200°K. The $T_r$ surface plots of the GRIN lens actuator design are presented in Fig 7. These surface plots were obtained using Equation (2). In the contour maps of Fig 7B, the flat region corresponds to the optimal angles of the $T_r$, therefore the optimal $T_r$ is moderately insensitive to variations in $\theta_1$ and $\theta_2$.

![Figure 7: Transmission ratio of the GRIN lens actuator for a low power objective](image)

The design parameters and simulated performance of the GRIN lens and prism actuators are summarized in Table 1.

<table>
<thead>
<tr>
<th>TABLE 1. Design parameters and performance</th>
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<td><strong>GRIN</strong></td>
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<td>$T_r$</td>
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<td>45.1</td>
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<tr>
<td>$\theta_1$</td>
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<td>$\theta_2$</td>
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<td>Stroke</td>
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<td>Power</td>
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<table>
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<th><strong>Prism</strong></th>
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<td>$T_r$</td>
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<tr>
<td>$\theta_1$</td>
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<td>$\theta_2$</td>
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<tr>
<td>Stroke</td>
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<td>Power</td>
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CONCLUSION

In this paper, we have presented a parametric model and the general design steps that may be used to design and optimize cascaded chevron flexures. These results enable designers to systematically obtain the optimal mechanical design for their applications. The results may be used within the overall optimization procedure of thermomechanical chevron actuators.

REFERENCES


