Field-of-View Distortion Calibration and Compensation for Ultra-high Accuracy Video-Based Metrology Systems
Yuhua Ding, Joe Tobison
Micro Encoder Inc.
Kirkland, WA, USA

ABSTRACT
Field-of-View (FOV) distortion affects measurement accuracy by generating location-dependent results in video-based metrology systems. For ultra-high accuracy systems equipped with high-quality lenses, this error can be as small as the uncertainty of obtainable dimensional references. In this research, we develop a method that calibrates the distortion using an uncertified planar artifact. Since the measurement is performed on a plane with a (nominal) constant magnification, we can decouple the calibration for distortion and the calibration for pixel scaling factors, eliminating the need for high-accuracy dimensional references. We first establish a distortion model, and then we solve the model parameters by nonlinear least squares fitting. Furthermore, we find the significant distortion terms by adding them one by one to the model and comparing the fitting residual and convergence. Experimental results of the method on five system configurations show reduced measurement variation across the FOV by up to 80%.

INTRODUCTION
A constant magnification across the Field-Of-View (FOV) is desirable in ultra-high accuracy (UHA) dimensional metrology systems equipped with video-probes and high-quality lenses. Such a system would ideally render true-to-scale images of the 2D object in the object plane, enabling one to apply constant scaling factors in converting pixel dimensions to physical dimensions.

Customers, however, often obtain slightly different measurement results when measuring the same feature in different regions of the FOV. This discrepancy arises from distortion caused by system imperfections such as optical aberrations and non-square sensor elements. In FIGURE 1, for example, the image of an equally spaced grid is no longer true-to-scale because of imaging system distortion. The distortion is nonlinear, which invalidates the assumption of constant magnification, resulting in FOV location-dependent measurement errors.

To maintain a constant scaling factor, we therefore need to calibrate and compensate the nonlinear distortion in order to recover the distortion-free, true-to-scale image of the object. This is a challenging task mainly due to the small magnitude of distortion in UHA systems (often in the sub-micrometer range).

Relation to previous work
Distortion calibration is normally performed as part of camera geometric calibration, where both intrinsic and extrinsic parameters are estimated by measuring features of known physical dimensions ([1,2]). Most artifacts, however, are only accurate to 1µm or 0.1µm, which is not sufficient for calibrating the submicron distortions for UHA systems. The motivation of this research is to develop a calibration method that does not rely on the knowledge of physical dimensions of the artifact.

Contribution of this paper
We exploit the fact that non-linear distortion can be decoupled from scaling factor and develop a method to calibrate the nonlinear FOV distortion using an uncertified planar Chrome-On-Glass (COG) artifact with features whose pixel locations can be measured accurately. Since the artifact does not need to be certified, its cost is greatly reduced. Furthermore, our method does not need a high-accuracy positioning system and is practical for on-site calibration.

DISTORTION CALIBRATION
Distortion model
The distortion model for UHA systems can be derived from the general pinhole camera model.
by assuming a constant nominal magnification.

A general pinhole model consists of four steps ([1,2]):

Step 1: 3D rigid body transformation from the World Coordinate System WCS \([x_w, y_w, z_w]^T\) to the Camera Coordinate System CCS \([x_c, y_c, z_c]^T\)

\[
\begin{bmatrix}
  x_c \\
  y_c \\
  z_c
\end{bmatrix} = R_{3 \times 3} \begin{bmatrix}
  x_w \\
  y_w \\
  z_w
\end{bmatrix} + T_{3 \times 1} \tag{1}
\]

Step 2: Projection from 3D CCS to undistorted Image Coordinate System ICS (for focal length \(f\))

\[
\begin{bmatrix}
  x_0 \\
  y_0
\end{bmatrix} = f \begin{bmatrix}
  x_c \\
  y_c
\end{bmatrix} \tag{2}
\]

Step 3: Radial and tangential distortions from undistorted image coordinates to real (distorted) image coordinates in the ICS

\[
\begin{bmatrix}
  x_d \\
  y_d
\end{bmatrix} = \begin{bmatrix}
  x_0 \\
  y_0
\end{bmatrix} + \begin{bmatrix}
  D_x \\
  D_y
\end{bmatrix} \tag{3}
\]

where

\[
\begin{bmatrix}
  D_x \\
  D_y
\end{bmatrix} = (k_1 r^2 + k_2 r^4 + k_3 r^6) \begin{bmatrix}
  x_0 \\
  y_0
\end{bmatrix} \tag{4}
\]

Step 4: Real image coordinate \([x_d, y_d]^T\) to the column and row \([X_f, Y_f]^T\) in the discretized image

\[
\begin{bmatrix}
  X_f \\
  Y_f
\end{bmatrix} = \begin{bmatrix}
  s_x & c_x \\
  0 & s_y
\end{bmatrix} \begin{bmatrix}
  x_d \\
  y_d
\end{bmatrix} + \begin{bmatrix}
  O_x \\
  O_y
\end{bmatrix} \tag{5}
\]

where \([O_x, O_y]\) is the column and row of the principle point (intersection of the optical axis and the image plane), \(c_x\) the shearing (nonperpendicularity) of \(x\) and \(y\) axes of the sensor, and \(s_x\) and \(s_y\) the scaling factors (pixels/mm) in \(x\) and \(y\), respectively.

In UHA systems, we assume that both the image and object planes are parallel to the focal plane (FIGURE 2). That is, \(z_c\) is constant. We normalize WCS and CCS by \(f/z_c s_x\) and ICS by \(1/s_x\) so that they are in pixel units, use capital letters for the normalized coordinates, and rewrite the four steps above as follows:

Steps 1 and 2 are combined into a 2D rigid body transformation from normalized WCS to normalized distortion-free ICS

\[
\begin{bmatrix}
  X_0 \\
  Y_0
\end{bmatrix} = R_{2 \times 2} \begin{bmatrix}
  x_c \\
  y_c
\end{bmatrix} + T_{2 \times 1} \tag{6}
\]

Step 3: Nonlinear distortion in normalized ICS

\[
\begin{bmatrix}
  X_d \\
  Y_d
\end{bmatrix} = \begin{bmatrix}
  X_0 \\
  Y_0
\end{bmatrix} + \begin{bmatrix}
  D_x \\
  D_y
\end{bmatrix} \tag{7}
\]

where

\[
\begin{bmatrix}
  D_x \tag{8}
  D_y
\end{bmatrix} = (k_1 r^2 + k_2 r^4 + k_3 r^6) \begin{bmatrix}
  x_0 \\
  y_0
\end{bmatrix} \tag{8}
\]

Step 4: Discretization of the image

\[
\begin{bmatrix}
  X_f \\
  Y_f
\end{bmatrix} = \begin{bmatrix}
  C_x & X_d \\
  0 & S_y
\end{bmatrix} + \begin{bmatrix}
  O_x \\
  O_y
\end{bmatrix} \tag{9}
\]

where \(C_x = c_x s_x, S_y = s_y\)

Since rotation and translation in the combined steps 1 and 2 do not distort the image, steps 3 and 4 define our final distortion model:

\[
\begin{bmatrix}
  X_f \\
  Y_f
\end{bmatrix} = M_{2 \times 2} \begin{bmatrix}
  x_0 \\
  y_0
\end{bmatrix} \tag{10}
\]

This model consists of the following terms:

<table>
<thead>
<tr>
<th>Term</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principle point</td>
<td>([O_x, O_y]) pixel</td>
</tr>
<tr>
<td>Pixel aspect ratio</td>
<td>(S_y) none</td>
</tr>
<tr>
<td>Shearing</td>
<td>(C_x) none</td>
</tr>
</tbody>
</table>
| Radial | \(k_1, k_2, k_3\) 1/pixel
| Tangential | \(p_1, p_2\) 1/pixel |

Since our model does not involve physical dimensions, distortion calibration is performed entirely in pixel coordinates without requiring certified feature dimensions.
**Calibration procedure**

The distortion can be calibrated using a planar target with well-defined features such as circles or corners of a checker board pattern on a COG artifact. Calibration is performed in three steps:

1. Place the artifact in the object plane (by focusing on the surface).
2. Move the set of features to different regions ("pose") of the FOV by in-plane translation and rotation. Measure the pixel location of each feature at each location (FIGURE 3).
3. Solve the distortion model using nonlinear Least Squares Fitting (LSF), as shown in FIGURE 4.

The fitting procedure solves for three types of variables:

1. The distortion model parameters
2. The distortion-free feature positions at the 1st pose
3. The 2D rigid body transformations of the target from the 1st pose to all other poses

Thus, the total number of fitting variables is:

\[
N_{\text{variables}} = N_{\text{model parameters}} + 2N_{\text{feature}} + 3(N_{\text{pose}} - 1) \quad (11)
\]

To solve for the fitting variables, we need at least \( N_{\text{variables}} \) data points:

\[
N_{\text{data}} \geq N_{\text{variables}}. \quad (12)
\]

Replacing \( N_{\text{data}} \) with \( 2N_{\text{feature}}N_{\text{pose}} \), we get

\[
(2N_{\text{feature}} - 3)(N_{\text{pose}} - 1) \geq N_{\text{model parameters}} \quad (13)
\]

For example, we can measure as few as 2 features in at least \( N_{\text{model parameters}} + 1 \) poses or as few as two poses containing at least \((N_{\text{model parameters}} + 3)/2\) features.

**Determination of significant terms**

Our distortion model in equation (10) captures the common distortion terms reported in the literature. For a specific optical system, however, only a few terms dominate. Many terms do not improve fitting and may in fact increase the sensitivity of the model and decrease the numerical stability of fitting. This is true especially for the higher-order radial and tangential terms ([1]).

To determine the significant distortion terms for a given system, we use different combinations of distortion terms in the model and compare the residual and convergence of the nonlinear LSF. We considered the models listed in TABLE 1. Model M0 is the simplest one, with the pixel aspect ratio term only. Model M0 serves as our baseline for comparison, because this term is already inherent in UHA systems that calibrate separate pixel scaling factors for \( x \) and \( y \). Starting from M0, we gradually add radial, shearing, and tangential terms. The 3rd-order radial term \((k_1)\) is added first since it is typically the dominating distortion.

**TABLE 1. Distortion models being tested.**

<table>
<thead>
<tr>
<th>Model</th>
<th>([O_x, O_y]), (k_1), (k_2), (k_3), (p_1, p_2), (C_X)</th>
<th>(S_Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M0</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>M1</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>M2</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>M3</td>
<td>✔️ ✔️</td>
<td>✔️ ✔️</td>
</tr>
<tr>
<td>M4</td>
<td>✔️ ✔️ ✔️</td>
<td>✔️ ✔️</td>
</tr>
<tr>
<td>M5</td>
<td>✔️ ✔️ ✔️ ✔️</td>
<td>✔️ ✔️</td>
</tr>
</tbody>
</table>

**EXPERIMENTAL RESULTS**

Accuracy evaluation on UHA systems is a challenging task due to the small magnitude of the distortion and additional system error sources besides distortion, such as positioning system errors and errors in pixel scaling factors.

A straightforward way to test the accuracy is to measure features of known dimensions. That is, the feature dimensions must be certified to an accuracy of about one magnitude higher than the distortion-related errors. For UHA systems, such an artifact is costly and can be difficult to obtain.

An alternative way of testing the accuracy is to check the measurement variation across the FOV. We choose this way because the goal of our research is to reduce location-dependent variations. The challenge, however, is to isolate
distortion from other error sources. The feature(s) to be measured need to meet two criteria:
1. Distortion-related error can be seen clearly from the measurements.
2. Distortion-related error dominates or can be easily separated from other errors.

For our experiments, we measure the diagonal distance of a square feature on an artifact moved to 9 regions of the FOV and oriented at 0- and 90-degree rotational angles (FIGURE 5). At each location, we extract the corners by a sub-pixel corner detection algorithm, apply distortion compensation, and compute diagonal distances (in pixel values). The Peak-To-Peak variation (P2P) of the 18 measurements represents distortion-related error. For each model M1 to M5, we perform calibration, apply compensation to get the P2P, and compute the improvement with regard to model M0 (FIGURE 6). Models with smaller P2P are considered to be more accurate.

TABLE 2 lists the configurations we tested and the corresponding improvement of the P2P variation of the diagonal distances. The test was performed on CNC vision measuring systems equipped with video probes. The method involves measuring the same set of uncertified but well-defined features on a planar artifact relocated to different regions of the FOV, then solving for the model parameters using nonlinear LSF. Test results of up to 80% smaller measurement variation show that the proposed method is effective in reducing measurement errors caused by camera distortion.

CONCLUSION
We developed a method to calibrate camera distortion for high-accuracy dimensional gauging systems equipped with video probes. The method involves measuring the same set of uncertified but well-defined features on a planar artifact relocated to different regions of the FOV, then solving for the model parameters using nonlinear LSF. Test results of up to 80% smaller measurement variation show that the proposed method is effective in reducing measurement errors caused by camera distortion.

REFERENCES