DESIGN AND OPTIMIZATION OF AN ACTIVE AEROSTATIC THRUST BEARING

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ABSTRACT
The properties of a circular aerostatic thrust bearing with an actively controlled flexible bearing surface are investigated. The flexible surface consists of a simply supported plate. Variation of the radius on which this plate is supported, results in a bearing with increased or reduced stiffness. In addition, in order to actively control the bearing an ideal force actuator to modify the bearing flexure is added. The static properties of the bearing are studied and furthermore it is shown that the dynamic properties of this type of bearing are primarily determined by the air film, and less by the dynamics of the flexible plate.

NOMENCLATURE
Dimensionless variables/ constants
- \( C_p \): pressure constant = \( p_s R^2 / E d h_{0,R} \)
- \( d \): bearing plate thickness = \( d/R \)
- \( D \): flexural rigidity = 1/2(1 + \( \nu \))
- \( G \): shear modulus = 1/12(1 – \( \nu \)^2)
- \( H \): film height = \( h/h_{0,R} \)
- \( h, \tilde{h} \): static, dynamic film height
- \( H_0 \): nominal film height = \( h_0/h_{0,R} \)
- \( \tilde{h}_0, \tilde{h}_0 \): static, dynamic nominal film height
- \( L \): bearing load
- \( M_r, M_0 \): radial and circumferential bending moment per unit length
- \( P \): pressure = \( p/p_a \)
- \( \bar{P}, \tilde{P} \): static, dynamic pressure
- \( Q \): transverse shear force per unit length
- \( r \): radial coordinate
- \( t_f \): specific time (air film) = \( \eta R^2 / p_s h_{0,R}^2 \)
- \( t_s \): specific time (plate) = \( (\rho R^2 / E) \nu \)
- \( W \): transverse flexure = \( w/h_{0,R} \)
- \( \bar{w}, \tilde{w} \): static, dynamic transverse flexure
- \( \Psi \): rotation of transverse normal
- \( \bar{\Psi}, \tilde{\Psi} \): static, dynamic rotation of transverse normal
- \( \kappa \): shear correction factor = \( \pi^2/12 \)
- \( \nu \): Poisson ratio
- \( \tau \): time

Parameters (values in reference point)
- \( d \): plate thickness = (0.005 m)
- \( E \): Youngs modulus = (2.1 · 10¹¹ Pa)
- \( h_{0,R} \): nominal film thickness = (10 · 10⁻⁶ m)
- \( p_a \): ambient pressure = (10⁵ Pa)
- \( p_s \): supply pressure = (5 · 10⁵ Pa)
- \( R \): bearing radius = (0.05 m)
- \( \eta \): air viscosity = (17.1 · 10⁻⁶ Pa s)
- \( \rho \): bearing density = (7800 kg/m³)

INTRODUCTION
The aerostatic thrust bearing is a bearing type commonly used in high precision linear and planar stages. In an aerostatic bearing a pressurized gas (usually air) is supplied between the bearing surfaces, thereby fully separating the surfaces and forming a lubricating film in between. The aerostatic bearing combines a simple design with excellent properties, i.e. low or zero friction, no stick-slip behavior, and high load capacity and stiffness. In a typical aerostatic bearing design, the air is supplied to the film via a number of supply restrictors. The load carrying capacity of the aerostatic bearing is proportional to the air pressure in the film at those supply points and the shape and height distribution of the lubricating film.

Bearing stiffness is defined as the gradient of the load carrying capacity with respect to the nominal film height. One method to ensure that an aerostatic bearing has sufficient passive bearing stiffness is to use a non-parallel film. In a bearing with a non-parallel film the pressure distribution will change as a result of a change in nominal film height, resulting in a normal stiffness. Holster e.a. [1] extended this basic idea of the non-parallel film to use the elastic flexure of the bearing as a result of the film pressure. An elastically supported, compliant plate bearing was developed, with the plate support positioned at a radius somewhere between the center and the edge of the bearing. It was shown that this bearing type exhibits a increased passive stiffness and
even an infinite stiffness for a small range in film height.

In order to further increase and in particular control the load carrying capacity and stiffness, active aerostatic bearings have been developed. In active aerostatic bearings either the supply pressure, or the resistance of the supply restrictors, or the bearing film height distribution is controlled. The third method, control of the bearing film height distribution, is less sensitive to delay times in the lubricating film and is to be preferred. Al-Bender e.a. [2] have investigated this type of actively controlled aerostatic bearing. In their design the central supply point in the compliant plate bearing surface was fixed, and the edge of the bearing is controlled using piezo-actuators.

The goal of this present study is to arrive at a bearing configuration that, depending on the actuating action, exhibits either very low stiffness, so the bearing can function as a vibration isolator, or very high stiffness, so the bearing can function to optimally support a vibrating machine. To that end, a combination of both these earlier configurations [1, 2] is presented (figure 1). The bearing geometry is similar to that of the bearing presented in [1]. A flexible plate is supported on an intermediate radius \( b \) and the bearing is supplied through a central supply point and recess \( a \). However in addition, the height distribution of the lubricating film is controlled using an ideal force actuator, positioned at the edge of the bearing \( c \). In contrast to [1] the central area of the bearing is not supported by the supply pressure.

In order to better explain the basic operation of this bearing a typical film height – load curve and its corresponding film height – stiffness curve are presented in figure 2 with the (schematic) film height and pressure distributions at three different nominal film heights presented in figure 1. For high film heights (point A, see also figure 1) the flexure of the bearing is negligible compared to the nominal film height. As a result the film shape and therefore pressure distribution and load carrying capacity are largely independent of the film height. The bearing has close to zero stiffness. For intermediate film heights (point B) the variation of pressure distribution for changing film heights is at a maximum, and the bearing has maximum stiffness. Finally, for very small film heights (point C) the pressure distribution approaches its asymptotic value and again the stiffness is small.

The properties of the bearing are determined by its geometry, in particular the position of the support point \( r_b \), and by the action of the actuator. In the next section, a mathematical model is presented to determine these properties.

**MATHEMATICAL MODEL**

In order to determine the characteristics of the bearing a mathematical model has been developed. The model describes both the aerostatic pressure distribution \( P \) in the lubrication film and the deformation of the bearing surface \( W \). The pressure distribution is calculated using the Reynolds’ equation and the flexure of the bearing surface is calculated using the Mindlin thick plate...
theory which describes the flexure and rotation of the neutral surface of the bearing plate.

It is assumed that the deformation of the bearing is restricted to the flexure of the plate. The support of the plate and the base of the bearing are assumed to be rigid. The support point is assumed to have no bending stiffness and the bearing plate is assumed to be simply supported. Furthermore, the bearing geometry is axisymmetric. So in our model the bearing geometry can be reduced to a one-dimensional domain: a line with coordinate \( r \) between 0 and 1. The central supply recess has a radius \( r_a \) and the bearing support is placed at a radius \( r_b \).

The model consists of three partial differential equations and their boundary conditions. The dimensionless pressure distribution \( P \) follows from the solution of the axisymmetric Reynolds’ equation:

\[
\frac{\partial}{\partial r} \left( r H^3 P \frac{\partial P}{\partial r} \right) = 12r \frac{\partial PH}{\partial \tau} \tag{1}
\]

Reynolds’ equation is valid for thin laminar lubricating films in which viscous forces dominate. The lubricating film height \( H \) is the sum of the nominal film height and the flexure of the bearing plate:

\[
H = H_0 + W \tag{2}
\]

The flexure of the bearing is described using the Mindlin thick plate theory (see e.g. [3]). In this case, the equations of motion for an axisymmetric loaded homogeneous circular plate are:

\[
\frac{\partial}{\partial r} \left( r Q \right) = -C_p r (P - 1) + r \left( \frac{t_s}{t_f} \right)^2 \frac{\partial^2 W}{\partial \tau^2} \\
\frac{\partial}{\partial r} \left( r M_r \right) = M_0 + \frac{r Q}{\bar{d}^2} + \frac{r}{12} \left( \frac{t_s}{t_f} \right)^2 \frac{\partial^2 \psi}{\partial \tau^2} \tag{3}
\]

where the force and moment resultants are given by:

\[
Q = \kappa G \left( \psi + \frac{\partial W}{\partial r} \right) \\
M_r = D \left( \frac{\partial \psi}{\partial r} + \frac{\nu \psi}{r} \right) \\
M_0 = D \left( \frac{\psi}{r} + \nu \frac{\partial \psi}{\partial r} \right) \tag{4}
\]

where \( W \) and \( \psi \) denote the dimensionless transverse deflection and rotation of a transverse normal.

The relative load carrying capacity \( L \) of the bearing is found using:

\[
L = \frac{2\pi}{L_{\text{max}}} \int_{r=0}^{1} r P \, dr \tag{5}
\]

where the load is given relative to the maximum obtainable load carrying capacity \( L_{\text{max}} = \pi (P_s - 1) \) and the dimensionless bearing stiffness \( S \) is defined as:

\[
S = -\frac{H_0}{L} \frac{\partial L}{\partial H_0} \tag{6}
\]

We are interested in both the static and dynamic properties of the bearing. To that end this model uses a two-step approach: First the stationary characteristics of the bearing are determined in a nominal operating point. Secondly, using a linearized perturbation technique the dynamic characteristics of the bearing (stiffness and damping) around this nominal point are determined. It is assumed that the solution \( P, W, \psi, H \) is the sum of a stationary solution \( \bar{P}, \bar{W}, \bar{\psi}, \bar{H} \) and a small dynamic and complex perturbation around this solution:

\[
P = \bar{P} + \bar{P} e^{j \omega \tau} \\
W = \bar{W} + \bar{W} e^{j \omega \tau} \\
\psi = \bar{\psi} + \bar{\psi} e^{j \omega \tau} \\
H = \bar{H} + \bar{H} e^{j \omega \tau} = (\bar{h}_0 + \bar{w}) + (\bar{h}_0 + \bar{w}) e^{j \omega \tau} \tag{7}
\]

STATIONARY PROPERTIES

Substituting eq. 7 in the in Reynolds’ equation 1 and plate flexure equations 3 and linearising around the operating point yields three coupled partial differential equations for the stationary operating solution. These equations can be solved, and the stationary properties of the bearing can be determined. To this end a commercial finite element code [4] is used. The pressure \( \bar{P} \), and flexure \( \bar{W} \) are approximated using cubic Lagrange elements, whereas the rotation \( \bar{\psi} \) is approximated using quadratic elements.

First the influence of the position of the support point is determined (figure 3). The support radius is varied between \( r_b = 0.9 \) (close to the bearing edge) until \( r_b = 0.3 \). Starting at \( r_b = 0.9 \) and reducing the support radius a clear trend can be found: The stiffness decreases steadily until for \( r_b = 0.5 \) it is found that the stiffness is close to zero for a large range of the operating conditions and the stiffness becomes negative for even smaller values of \( r_b \). Furthermore it is clear
that the optimal film height, that is the film height where a maximum stiffness is reached, moves to higher values for an increase in $r_b$.

Next the properties of this bearing are studied for a prescribed actuator action. The actuator is placed at the edge of the bearing, and $r_b$ is chosen to be equal to 0.5. In figure 4 the load carrying capacity and required actuator force are presented, both normalized with the maximum obtainable load $L_{\text{max}}$. From this graph it is clear that any load carrying capacity within the theoretical limits is obtainable. However, the required actuator load varies between zero and five times the maximum theoretical bearing load.

**DYNAMIC PROPERTIES**

From the perturbed solution in eq. 7 the dynamic properties can be calculated as function of frequency. The frequency response is calculated both for a vibrating counter surface and for a vibrating bearing. From the results it is clear that only at high frequencies there is a clear difference between both responses. The dynamics of the plate bearing only come into play at these high frequencies.

**REFERENCES**


