ABSTRACT
In this paper we introduce a new approach – Freedom and Constraint Topology – to the conceptual design of parallel precision flexures. The import of this work is that it provides a means (a) to synthesize flexures with coupled or non-coupled degrees of freedom in a deterministic manner and (b) to rapidly generate flexure concepts with coupled or non-coupled motions. This method represents the general shape of a flexure’s constraining elements (constraint topology) and the flexure’s degrees of freedom (freedom topology) as 3D shapes. For a desired set of motions, there is a unique constraint topology. This geometric mapping technique makes it possible to select the proper constraint topology for a set of desired motions requirements. Several specific concepts may then be generated from the general 3D shape that represents the constraint topology.

INTRODUCTION
The intent of this paper is to introduce a new method – Freedom and Constraint Topology (FACT) – that may be used to synthesize multi–degree of freedom (MDOF) flexure system concepts. Flexure systems consist of a combination of rigid and flexural elements. These elements are arranged and connected in a way such that their compliant directions permit specified motions and their stiff directions prevent motions in other directions.

Flexures are important to conventional precision applications, for instance they are commonly used within optical manipulation stages, precision motions stages, nanomanufacturing equipment and instruments that are used in nano–scale research. Many of these devices require the capability to move in several axes such that the motions achieved via some axes may be coupled and/or they may be independent of the other motions. This paper provides a means to synthesize parallel flexure systems for these types of applications.

The synthesis of multi–axis parallel flexure concepts is difficult because there are several flexural components and these components provide constraints and permit motions in orientations that are not generally parallel between the different elements. It is necessary, and difficult, for designers to keep track of (a) the relative, three dimensional orientations of the flexural constraints, (b) the orientation of the permitted motions, and (c) the three dimensional relationships between each constraint and the permissible motions. Given the preceding, it should not be surprising that parallel flexure systems with three to five degrees of freedom (DOFs) are designed via iteration and designers are fortunate if they are able to synthesize one or two new and viable concepts.

Figure 1A provides a contrast of the traditional design method used in precision engineering, constraint–based design (CBD), and FACT.
In CBD, a designer must use his visualization skills, pattern recognition and CBD’s rule of complementary patterns to guide a visual iteration process until a viable flexure system concept is identified. Additional concepts are generated via more iteration. In contrast, FACT uses sets of 3D geometric shapes, for example planes and spheres, to embody quantitative information about a flexure system’s shape and its DOFs. These types of geometric shapes, for instance the examples shown in Figure 1Bii, are capable of displaying the general form of a flexure system design. All possible concepts are represented within the shapes and therefore a designer may have access to all solutions for a given motion guidance problem. These shapes may also be used to determine when coupled rotations–translations exist and they may also be used to visualize coupled rotations–translations.

In the FACT method, a first set of shapes contains information about the flexure system’s DOFs (its freedom topology) and a second set contains information about the flexure system’s geometry (its constraint topology). If a designer wanted to generate concepts for a flexure system with one rotation DOF, they would first specify the geometric shape (freedom topology) that represents this rotation. The designer then identifies the rotation line in Figure 1Bi as the desired topology. The principle of complementary topologies is then used to find the set of geometric shapes, i.e. the intersecting planes in Figure 1Bii, that represents concepts for all parallel flexure systems that can permit the specified DOF. The designer then selects constraints that lie within the shapes that represent permissible constraints in order to form various different concepts. For instance the three concepts shown in Figure 1Biii contain flexural constraints that lie within the planes. This example demonstrates the use of FACT for a simple flexure design, however more complex, multi-DOF flexures may be designed [1,2].

Designers need to understand two main issues with respect to FACT, the:

(1) geometric shapes that are used to synthesize parallel flexures and

(2) the process one should follow to use the shapes while forming concepts.

The details on the second point are summarized in a complimentary paper within these proceedings: A Design Process for the Creation of Precision Flexure Concepts via the use of Freedom and Constraint Topology. The contents of that document will be best appreciated after learning about the first point, the shapes, in this document.

BACKGROUND KNOWLEDGE USED IN FACT
This section provides an overview of principles that are central to the use of FACT.

Maxwell’s Principles of Constraint
A first principle of constraint–based design is that a rigid body has six DOFs and any non–redundant constraint upon the body removes a DOF. This may be expressed as:

\[ R = 6 - C \]  

where \( C \) is the number of non–redundant constraints and \( R \) is the number of DOFs. Non–redundant constraints are mathematically equivalent to constraints that possess lines of actions, i.e. vectors, which are independent.

A constraint is idealized as providing resistance to motion along its line of action only. Maxwell augmented this equation with observations that enable one to understand some of the DOFs that are permitted given the lines of action of a system of constraints [3]. Maxwell used these principles to design many types of precision instruments and fixtures for his experiments.

Projective Geometry
In the design of multi-axis devices, i.e. as in robotics, it is useful to visualize geometric shapes that possess a mix of finite dimensions and dimensions that approach infinity. The mathematic field of projective geometry addresses these types of geometric shapes. The first principle of import from this field may be stated as lines emulate circles with a radius of curvature that approaches infinity. The relevance of this principle is demonstrated in Figure 2. For small motions, translations may be emulated by rotations about a circle whose radius approaches infinity. The circle is shown as a “hoop” in Figure 2. The rotation of the stage could occur about points on this hoop such that the rotation yields a motion that emulates a translation in a direction that is perpendicular to the plane of the hoop. This is
an important concept because CBD and FACT treat translational motions as though they are the result of a rotation about a hoop.

The second relevant principle from this field is that parallel lines may be considered to share an intersection in the limit as the lines approach infinity. These principles have been used in CBD and will also be used in FACT to identify the intersections between geometric shapes that represent constraints and DOFs.

**The Rule of Complementary Patterns**

Blanding [4] viewed constraints and DOFs via constraint lines and freedom lines respectively. A constraint line is the line of action of an idealized constraint. All DOFs are viewed as rotations about a freedom line. The hoop principle from the previous section is used to describe translations in terms of rotations. Blanding’s Rule of Complementary Patterns [4] states that every freedom line intersects every constraint line. This is powerful because it enables a designer to visualize the relative relationships between a flexure system’s constraints and the DOFs that these constraints permit. The rule of Complementary Patterns has been used to design many mechanical devices, flexures and fixtures.

**Screw Theory**

To this point we have only considered rotation DOFs, i.e. freedom lines. It is possible for a parallel flexure to undergo coupled translations-rotations, i.e. screw lines. CBD principles cannot be used to diagnose or synthesize flexures that exhibit this type of behavior.

Screw theory may also be used to create geometric shapes that contain information about coupled DOFs. Screw theory may be used to derive the following relationship that links constraints and coupled rotations–translations:

\[ p \cdot \cos(\psi_i) = r_i \cdot \sin(\psi_i) \]  

(2)

In Equation 2, \( r_i \) is the shortest distance between the constraints and the axis of the screw motion, \( \psi_i \) is the skew angle between the screw and the constraint lines, and \( p \) is the translation per unit rotation, or pitch. In FACT, the shapes are used to visualize all permissible motions (rotations, translations and screws). The utility of the preceding will become apparent in a moment.

**PRINCIPLE OF COMPLEMENTARY TOPOLOGIES**

The Rule of Complementary Patterns that is used in CBD can not address coupled degrees of freedom and so it is not possible to use this principle alone if one wishes to understand/design for translations, rotations and screws. With the Principle of Complementary Patterns, it is possible to design a flexure that will end up possessing unwanted, coupled degrees of freedom. Given the necessity for deterministic design in precision engineering, it is desired to have a general principle that applies to coupled degrees of freedom.

The Principle of Complementary Topologies was created to map freedom and constraint topologies that are relevant when any combination of translations, rotations or screws occur. The principle states that a set of 3D shapes which contains freedom lines and a set of 3D shape that contain constraint lines are complementary when all the constraint lines are complimentary to all the freedom lines. This condition occurs when Eq. 2 is satisfied for each constraint-freedom line pair between the sets.

In practical terms, this principle (a) enables the matching of any set of desired motions (translations, rotations and screws) with shapes wherein any of the constraint lines is a valid option for use in the flexure and (b) may be used to prove [1] that there are only 26 different sets of motions (translations + rotations + screws) and 26 different types of parallel flexures that provide them. This is good news for designers as they only need to learn how to translate their motions’ requirements into the shapes that represent the motions and then find the matching set of shapes that show them how to arrange a flexure’s geometry to allow the desired motions. It is not possible to show them here due to size limitations. The proof of this statement, and the 26 types of parallel flexures, are available in the reference [1,2].
USING FACT TO GENERATE CONCEPTS
As an example (see [2] for more examples) suppose we desired to synthesize concepts for a parallel flexure (not a stacked set of flexure stages) that would translate and rotate an optic. The translation must be orthogonal to the axis of rotation and all other motions are to be resisted. The sets of shapes that represent the desired motions are shown in Figure 3A and the complementary set of shapes that contain permissible constraints are shown in Figure 3B.

The former consists of (1) a hoop that represents the translation and (2) a plane of parallel freedom lines that are orthogonal to the direction of the translation. The latter consists of (1) a box that represents every line that is orthogonal to the translation and (2) every line that lies on a plane that is parallel to, or coincident with, the parallel constraint lines. We use Eq. 1, which tells us to select four non-redundant constraints from the box and plane in order to achieve two degrees of freedom.

Figures 3C–3F, 3G–3J, 3K–3N show three ways to pick four independent constraints. There are many other possible solutions that may be generated using these shapes.

REFERENCES


FIGURE 3. The freedom space for orthogonal rotational and translation motions (A) and the constraint space (B) that yields several viable topologies/embodiments – (C), (G), and (K). FEA post processing of the components – (D), (H), and (L) – of each embodiment contrast the respective deformed and non-deformed states for the rotational motion – (E), (I), and (M) – and the translation motion – (F), (J), and (N) – of each flexure.