INTRODUCTION
Nanotechnology and ultra-precision engineering have progressed very rapidly. Displacement measurements with less than sub-nanometer resolutions have become essential and critical. A scanning tunneling microscope (STM) [1] is a popular tool in surface engineering and can be used to obtain images of atoms on a crystalline surface. A graphite crystalline surface has uniform and stable 2-dimensional symmetrical periodicity, i.e., two unit lattice vectors \( a_1 \) and \( a_2 \), whose length and intersectional angle are 0.246 nm and 60° [2], respectively, over a wide mono-crystalline area of approximately 100 \( \mu \)m², when the crystal is stress-free. Therefore, the two unit lattice vectors of the graphite crystalline surface can be used, in combination with an STM, as 2-dimensional references with sub-nanometer resolution or less [3-4]. In this article, we propose a new 2-dimensional encoder using multi-tunneling-probes scanning tunneling microscope (MTP-STM) as detectors and the unit lattice vectors \( a_1 \) and \( a_2 \) on a graphite crystalline surface as references. The measurement principle, the instrumentation and the experimental results are discussed.

PRINCIPLES & INSTRUMENTATION
Three points method
Figure 1 shows an ideal atomic image of the graphite crystalline surface. In figure 1, \((a_1, a_2)\) and \((k_1, k_2)\) are the unit lattice vectors between an \( \alpha \) sites [2] and their reciprocal vectors, respectively. The unit lattice vectors and the reciprocal vectors have the following relationship,

\[
a_i \cdot k_j = \delta_{ij} \quad (i, j = 1, 2) \quad \text{Kronecker’s } \delta. \quad (1)
\]

In the figure 1, \( k_i = k_i^* \). Topographical height \( Z = Z(r) \) at an arbitrary lateral position \( r \) can be represented approximately as:

\[
Z(r) = \sum_{i=1}^{3} A_i \cos(2\pi k_i \cdot r), \quad (2)
\]

where \( A_i \) (i = 1, 2, 3) is a contrast parameter for the atomic structure. In figure 1, we assume that three STM probes (tips) are located properly on the three points \((A, B, C)\) which satisfy the following relationship:

\[
r_A = ma_1 + na_2, \quad (3)
\]

\[
r_B = (m + \frac{1}{2})a_1 + na_2,
\]

\[
r_C = ma_1 + (n + \frac{1}{2})a_2. \quad (4)
\]

In the equations (3) - (4), \((m, n)\) is a set of real numbers to show the position of the representative point A. The vector \( r_A \) also represents the relative position (displacement) vector between the multi-tips and the crystalline surface on the XY sample stage. From equations (1) - (4), we can obtain the following two relationships:

\[
Z(r_A) + Z(r_B) = 2A_1 \cos 2\pi m
\]

\[
Z(r_A) + Z(r_C) = 2A_1 \cos 2\pi n. \quad (5)
\]

If we locate the multi-STM tips properly on the points \((A, B, C)\), a set of \((m, n)\) can be determined using equation (5). This means that a 2-dimensional displacement using linear sums of the multi-tunneling-current signals of the points \((A, B, C)\) is possible. To obtain the linear height \( Z(r) \) from the tunneling currents, the use of log amplifiers for each STM tip is suggested. In the paper, the method to determine \((m, n)\) using equation (5) is called as the three points method. The points \((A, B, C)\) is called as the three points group.

Eight points method
Figure 2 shows the locations of the multi STM tips on a double crystalline area of the graphite crystal. In the figure, the unit lattice vectors of the two crystalline areas \( S\alpha \) and \( S\beta \) are slightly different. However, the two crystalline areas are set on the
same XY sample stage, and the averaged planes of each crystalline area must be parallel. In figure 2, \((a_1, a_2)\) and \((a'_1, a'_2)\) show unit lattice vectors for the two crystalline areas, and \((\alpha, \beta)\) are intersection angles of \((a_1, a'_1)\) to the X-axis of the sample stage. The unit lattice vectors are represented as:

\[
\begin{align*}
a_1 &= d \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix}, \\
a_2 &= d \begin{pmatrix} \cos(\alpha + \frac{\pi}{3}) \\ \sin(\alpha + \frac{\pi}{3}) \end{pmatrix}, \\
a'_1 &= d \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix}, \\
a'_2 &= d \begin{pmatrix} \cos(\beta + \frac{\pi}{3}) \\ \sin(\beta + \frac{\pi}{3}) \end{pmatrix}.
\end{align*}
\]

In figure 2, points A and E are representative points and vectors \(r_A\) and \(r_E\) show the position (displacement) vectors of the sample stage \((r_A = r_E)\). The position vectors \(r_A\) and \(r_E\) can be written as:

\[
\begin{align*}
r_A &= m_1a_1 + n_1a_2, \\
r_E &= m_2a'_1 + n_2a'_2.
\end{align*}
\]

where \((m_1, n_1)\) and \((m_2, n_2)\) are sets of real numbers.

From equations (6) and (7), we obtain:

\[
r = r_A - r_E = \sqrt{\frac{1}{2} \sum_{i=1}^{2} \left[(m_i - n_i) \left(a_i + a'_i\right)\right]^2}.
\]

Once \((n_i, n_2)\) can be obtained in the case \(\beta\) does not equal \(\alpha\), then the displacement vector \(r = r_A - r_E\) of the sample stage can be determined. In figure 2, we assume the eight STM tips are located properly at the eight points \((A, B, C, D)\) and \((E, F, G, H)\) satisfying equation (7) and the following conditions:

\[
\begin{align*}
r_B &= (m_1 + \frac{1}{2})a_1 + n_1a_2, \\
r_C &= (m_1 + \frac{1}{2} + \frac{1}{3})a_1 + (n_1 - \frac{1}{3})a_2, \\
r_D &= (m_1 + \frac{1}{2} + \frac{1}{3})a_1 + (n_1 - \frac{1}{3})a_2, \\
r_E &= (m_2 + \frac{1}{2})a'_1 + n_2a'_2, \\
r_F &= (m_2 + \frac{1}{2} + \frac{1}{3})a'_1 + (n_2 - \frac{1}{3})a'_2, \\
r_G &= (m_2 + \frac{1}{2} + \frac{1}{3})a'_1 + (n_2 - \frac{1}{3})a'_2, \\
r_H &= (m_2 + \frac{1}{2})a'_1 + (n_2 - \frac{1}{3})a'_2.
\end{align*}
\]

From equations (2), (7) and (9), the following results are obtained:

\[
\begin{align*}
Z(r_A) + Z(r_B) &= 2A_1 \cos 2\pi m_1, \\
Z(r_C) + Z(r_D) &= 2A_1 \sin 2\pi m_1, \\
Z(r_E) + Z(r_F) &= 2A'_1 \cos 2\pi m_2, \\
Z(r_G) + Z(r_H) &= 2A'_1 \sin 2\pi m_2.
\end{align*}
\]

Equation (10) shows that linear sums of multi-tunneling-current signals from points \((A, B, C, D)\) and \((E, F, G, H)\) can be used to derive \((n_1, n_2)\) with its fractional part, i.e., the two-displacement of the sample stage can be determined with interpolation less than the lattice spacing. In the paper, the method to determine \((n_1, n_2)\), i.e., the displacement vector \(r\) of the sample stage, is called as the eight point method. The points \((A, B, C, D)\) or \((E, F, G, H)\) is also called as the four points group.

**Figure 2** Two ideal atomic images of the graphite crystalline surface, whose crystalline orientation in the two images are not same. \((a_1, a_2)\) and \((a'_1, a'_2)\) show unit lattice vectors for the two crystalline areas \((a)\) \(S\alpha\) and \((b)\) \(S\beta\)

**Semi-real time multi-probes**

It is very difficult to align multi-STM tips at the proper multiple points with an accuracy of less than the lattice spacing in the three points method or the eight points method. We propose a different method to obtain multi-current-signals from multi-points using a fast lateral dither vibration applied to one STM tip. **Figure 3** shows trajectories of high-speed lateral dither vibrations applied to an STM tip for the three points group and the four points group, respectively. In figure 3 (a), to obtain tunneling currents from three points \((A, B, C)\), a circular dither modulation is applied to an STM tip, and the data sampling interval is set to 120° in phase. In figure 3 (b), to obtain tunneling currents from the four points \((A, B, C, D)\) or \((E, F, G, H)\), an ellipsoidal dither modulation, whose major axis is parallel to the A-B and (E-F) line, is applied to an STM tip, and data sampling interval is set to 90° in phase.

**Figure 3** Trajectories of lateral dither modulation applied to an STM tip. In the figures, the lateral axis is assumed to be the crystalline orientation.
Instrumentation
In order to evaluate the proposed methods, a double-tunneling-probes scanning tunneling microscope (DTP-STM) was constructed, as shown in figure 4. The DTP-STM includes a base unit, a scanner unit, two STM heads with two STM tips, and a capacitance sensor. A tube type XY piezo-actuator is used as a sample scanner. A graphite crystal [5] is set on the sample scanner. The two STM heads are symmetrically placed with respect to the scanner. The STM heads have XYZ tube piezo-actuators as tip scanners for the Z-axis servo and high speed lateral dither modulation. Each STM head scans the three points group or the four points group using the lateral dither modulation. The capacitance sensor [6] is attached to the scanner unit to measure the displacement of the sample scanner. The measurement direction of the capacitance sensor is fixed at 45° to the X-axis of the sample scanner. Improvement of the resolution of 0.1 nm for the capacitance sensor was achieved with low pass filtering amplifier.

Figure 4 Double-tunneling-probes scanning tunneling microscope (DTP-STM). (a) Photograph of the DTP-STM. (b) Top view of a sample XY scanner unit.

EVALUATION EXPERIMENTS
Three points method
The three points method, wherein one STM tip is modulated with a circular dither vibration, was evaluated. The amplitude and the frequency of the dither modulation were 0.29 nm and 5.13 kHz, respectively. In the experiment, driving signals of reciprocal motion with an amplitude of approximately 40 nm and a frequency of 1.25 Hz along the 45° direction to the X-axis were applied to the scanner, and the derived displacement using the three points method was compared with the capacitance sensor. The whole measurement contains 20 reciprocal lines. Figure 5 shows the displacement measurement comparison between the three points method and the capacitance sensor. Figure 5 (a) shows the whole displacement from the 1st to the 20th lines. Figure 5 (b) shows time variations of the difference output of the three points method and the capacitance sensor at 1st and the 20th lines after thermal drift compensation. From figure 5 (b), the time variations of the displacement at the 1st and 20th reciprocal lines coincided within a dispersion of 0.2 nm. The tip speed was around 100 nm/s.

Figure 5 Comparison of displacement measurement between the three points method and the capacitance sensor. (a) Comparison of whole displacement measurement (from 1st to 20th reciprocal lines). (b) Difference between at the 1st and 20th reciprocal lines after thermal drift compensation.
**Eight points method**
The eight points method was also evaluated through comparison with the capacitance sensor. The lateral ellipsoidal dither modulations (amplitude of major and minor axes was 0.35 nm and 0.15 nm, frequency was 5.0 kHz) were applied to the two STM tips. In the experiment, driving signals of reciprocal motion with the amplitude of approximately 40 nm and frequency of 1.22 Hz along the 45° direction to the X-axis were applied to the XY scanner. The measurement result contained 20 reciprocal lines. **Figure 6** shows the displacement measurement comparison between the eight points method and the capacitance sensor. Figure 6 (a) shows the whole displacement from the 1st to the 20th lines. Figures 6 (b) shows the time variation of the difference between the output of the eight points method and the capacitance sensor at the 1st and 20th lines after thermal drift compensation. From figure 6 (b), the time variations at the 1st and 20th lines are coincident within a dispersion of 0.07 nm. Figure 6 (b) also proves that the difference between the proposed method and the capacitance sensor is approximately 0.6 nm (1.5%). The results show that the eight points method has the feasibility for 2-dimensional displacement measurement with a resolution or uncertainty on the order of 10 pm.

**CONCLUSIONS**
In the paper, we propose a two-dimensional (2-D) encoder with picometer resolution using the MTP-STM as detectors and the graphite crystalline lattice as a reference. We also propose a fast lateral dither vibration applied to one STM tip to obtain multi-current-signals from multi-points in semi real-time way. The proposed methods were compared with a capacitance sensor. The experimental results show that the proposed method has the capability for measuring 2-D lateral displacement with an uncertainty on the order of 10 pm with a maximum measurement speed of 100 nm/s or more.

**ACKNOWLEDGEMENTS**
Partial finance support of the Scientific Research Fund of the Japanese Ministry of Education, Culture, Sports and Technology is gratefully acknowledged.

---

**Figure 6** Comparison of displacement measurement between the eight points method and the capacitance sensor. (a) Comparison of whole displacement measurement (from 1st to 20th reciprocal lines). (b) Difference at the 1st and 20th reciprocal lines after thermal drift compensation.

**REFERENCES**
[6] ADE corporation, Microsense, USA.