NON-STATIONARY SYSTEMATIC ERROR DETECTION WITH EXPERIMENTAL VERIFICATION ON AN OPTICAL PROFILER

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Introduction

Variable or dynamic systematic errors existed in many measurement processes. It is caused by deviations or changes in measurement conditions. It is very difficult to detect for removal because stable measurement conditions are not easy to be maintained in many practical applications. One example is temperature drift. It occurs in many measurement devices involving electronic circuits because the temperature in the instruments will typically keep increasing for a certain period of time after power being switched on. For the cases where dynamic systematic errors exist together with random errors, the changes in system conditions are hard to notice. This will make the detection more difficult. In many cases, the effects due to dynamic systematic errors are ignored and the reliability of measurement results seriously affected. Measurement errors of the kind may be classified into two types, the stationary type and the non-stationary type. For the stationary type, systematic errors may be identified and reduced through a number of comparative experiments. For the non-stationary type, suitable data treatment methods may be applied. A number of statistical methods have been proposed to solve the problem, such as curve fitting data evaluation coupled with a statistical sensitivity analysis, Markov estimator, and residual error [1-3]. However, most of the methods are highly dependent on the use of statistics that inherently require the sampling size to be large and the distribution to be known in advance. For the cases of small sampling size and unknown distribution, the methods may become ineffective. A new dynamic systematic error detection method based on fuzzy set theory was proposed [4]. The new method works effectively when the sampling size is small and the sample distribution is unknown. Mathematical models and dynamic systematic error detection criteria are presented. In order to verify the effectiveness of the proposed method, a series of experiments were conducted on a precision optical profiler. Changes in system conditions, such as warm up, focus, magnification, measurement location, and temperature drift during a measurement process in the optical profiler were made and the effects of the conditions were examined.

Error Detection Model

Assume that there are $m$ quantities measured in measurement processes. In each of the process, an equal number of samples, noted as $n$, are measured. These set of data can form a measurement data matrix, noted as $Y$, as shown in Eq. (1).

$$
Y = \begin{pmatrix}
y_{11} & y_{12} & y_{13} & \cdots & y_{1n} \\
y_{21} & y_{22} & y_{23} & \cdots & y_{2n} \\
y_{31} & y_{32} & y_{ik} & \cdots & y_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
y_{m1} & y_{m2} & y_{m3} & \cdots & y_{mn}
\end{pmatrix}, \quad (1)
$$

where $y_{ik} \in \mathbb{R}$, $i = 1, 2, \ldots, m$, and $k = 1, 2, \ldots, n$. $\mathbb{R}$ is a real number set. After normalization, a new data set $X$ is obtained and it is a function of the original measurement data set $Y$, $X = f(Y)$. The normalization should not change the characteristics of errors in the original data [4]. The relation $R$ can be considered as
\[ R = \sum_{(x_i, x_l) \in R} \mu_R(x_i, x_l) / (x_i, x_l), \]  

(2)

where \( i, l = 1, 2, \ldots, m \). The term \( \mu_R(x_i, x_l) \) is the fuzzy membership grade of any two quantities \( x_i \) and \( x_l \) in the measurement data matrix. It is in the relation subset \( R \) of the real number set \( \mathbb{R} \), \( R \subset \mathbb{R} \). The relation can also be represented as \( R = [r_{il}, i = 1, 2, 3 \ldots m, l = 1, 2, \ldots, m] \), where \( r_{il} = \mu_R(x_i, x_l), i, l = 1, 2, \ldots, m \), and \( (x_i, x_l) \in \mathbb{R}^2 \). The matrix \( R \) can be used to represent the degree of associations among the quantities. A suitable model should be chosen to obtain each of the membership grades \( r_{il} \) \[4\]. In considering a simpler case, a min-max model is utilized as

\[
r_{il} = \frac{\sum_{k=1}^{n} (x_{ik} \land x_{lk})}{\sum_{k=1}^{n} (x_{ik} \lor x_{lk})},
\]

(3)

where \( \land \) and \( \lor \) are the fuzzy operations representing the minimum operation and the maximum operation respectively. The membership grade matrix \( R \) is symmetric as \( r_{il} = r_{li} \). The diagonal of the matrix is equal to 1 when \( i = l \).

The relationship between any one of the \( m \) quantities and the remaining \( m-1 \) quantities can be assessed by obtaining a transitive closure matrix \( T \). It is obtained by applying max-min closure of \( R \) \[4\]. The closure matrix is symmetric and the diagonal is equal to 1 (Eq. (4)).

\[
T = \begin{pmatrix}
1 & v_{12} & v_{13} & \ldots & v_{1l} & \ldots & v_{1m} \\
v_{12} & 1 & v_{23} & \ldots & v_{2l} & \ldots & v_{2m} \\
v_{13} & v_{23} & 1 & \ldots & v_{3l} & \ldots & v_{3m} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_{1l} & v_{2l} & v_{3l} & \ldots & 1 & \ldots & v_{lm} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
v_{1m} & v_{2m} & v_{3m} & \ldots & v_{lm} & \ldots & 1
\end{pmatrix},
\]

(4)

where \( 0 \leq v_{il} \leq 1 \), and \( i, l = 1, 2, \ldots, m \). Each element, \( v_{il} \), in the transitive closure matrix \( T \) represents the fuzzy equivalence relation or the similarity relation between two quantities, \( y_i \) and \( y_l \). If the value of the element \( v_{il} \) is close to 1, this represents the two quantities have a stronger level of association and the possibility of having systematic errors is low. However, if the value of the element \( v_{il} \) is close to 0, this means the two quantities have a weaker level of association and the systematic errors are existed.

**Error Detection Criteria**

Having obtained the transitive closure matrix \( T \), the detection of systematic errors can be done by comparing the elements \( v_{il} \) with an optimum level, \( \lambda \). The level was chosen to be \( \lambda = 0.5 \) because it is the average value of the minimum level of association, \( \lambda = 0 \), and the maximum level, \( \lambda = 1 \). The systematic errors exist between the quantities \( y_i \) and \( y_l \) if the value of the elements \( v_{il} \) is smaller than or equal to \( \lambda \), and vice versa \[4\]. In order to compare the elements with the optimum level, a \( \lambda \)-cut subset matrix \( R_\lambda \) is defined as

\[
R_\lambda = [r_{2il}],
\]

(5)

where \( r_{2il} = \begin{cases} 1, & v_{il} \geq \lambda \i, l = 1, 2, \ldots, m. \\
0, & v_{il} < \lambda \i, l = 1, 2, \ldots, m. 
\end{cases} \]

**Experimental Results and Discussion**
In order to verify the effectiveness of the method [4], several experiments conducted on a Veeco Wyko NT3300 optical profiler (Fig. 1) in PSI and VSI modes were performed. It can measure heights from angstroms to millimetres. The vertical resolution is up to 3Å. The measurement samples are shown in Fig. 2-3.

![Fig. 1 Optical profiler](image1)
![Fig. 2 Testing sample for PSI](image2)
![Fig. 3 Testing sample for VSI](image3)

Fig. 1 Optical profiler  Fig. 2 Testing sample for PSI  Fig. 3 Testing sample for VSI

Totally 6 cases of conditions were examined (Table 1). In each case, 6 data sets with 5 data points in each of the data sets were collected. Based on the models (Eq. (1)-(5)), for each of the 6 cases, the measurement data matrix \( Y \), the transitive matrix \( T \), and the \( \lambda \)-cut matrix \( R_\lambda \) for both VSI and PSI modes are determined.

The results of \( Y \), \( T \), and \( R_\lambda \) for the change of location condition are presented. The variations in \( Y \) for the case are not large (Fig. 4) and it would be difficult to determine the existence of systematic errors.

\[
Y = \begin{bmatrix}
1.92 & 1.92 & 1.92 & 1.92 & 1.92 \\
1.92 & 1.92 & 1.92 & 1.92 & 1.92 \\
1.95 & 1.95 & 1.95 & 1.95 & 1.95 \\
1.97 & 1.97 & 1.97 & 1.97 & 1.97 \\
1.94 & 1.96 & 1.96 & 1.96 & 1.96
\end{bmatrix}, \quad
T = \begin{bmatrix}
1.00 & 1.00 & 0 & 0 & 0 \\
1.00 & 1.00 & 1.00 & 0.72 & 0.74 \\
0 & 0 & 1.00 & 1.00 & 0.72 \\
0 & 0 & 0.72 & 0.72 & 1.00 \\
0 & 0 & 0.74 & 0.74 & 0.72 & 1.00
\end{bmatrix}, \quad
\text{and } R_\lambda = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1
\end{bmatrix}.
\]

![Fig. 4 Measurement results under the change of location condition](image4)

Fig. 4 Measurement results under the change of location condition

For this case (Fig. 4), the minimum value of the elements in the transitive closure matrix \( T \) was 0. The value was much smaller than \( \lambda = 0.5 \). The minimum value of the elements in \( R_\lambda \) had zero values. Therefore, systematic errors were identified in the measurement data set (Fig. 4).

Table 2 shows the minimum value of the elements in the \( \lambda \)-cut matrix \( R_\lambda \). If it is 0, the systematic errors would exist. It was found that two conditions, the change of magnification and the change of measurement location, in the PSI mode were the most significant involving systematic errors, since more zero value elements in \( R_\lambda \) are observed (Table 2). It can be seen that for the VSI mode, there were two failures (Table 2). This could be due to the measurement...
resolution in the VSI mode being not high and the changes introduced in the two cases being not sufficiently large. However, the proposed method [4] could successfully determine if systematic errors exist for case of no condition change and for the five cases in the PSI mode (Table 2).

Table 1 Experimental conditions examined

<table>
<thead>
<tr>
<th>Condition</th>
<th>Condition changes</th>
<th>No. of repetition of a state $N_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No change</td>
<td>None</td>
<td>6</td>
</tr>
<tr>
<td>Change of magnification</td>
<td>10x, 20x, 50x</td>
<td>2</td>
</tr>
<tr>
<td>Change of measurement location</td>
<td>Three arbitrary positions</td>
<td>2</td>
</tr>
<tr>
<td>Focusing</td>
<td>Three arbitrary trials of focusing</td>
<td>2</td>
</tr>
<tr>
<td>Warm up</td>
<td>Switch on and off the machine</td>
<td>3</td>
</tr>
<tr>
<td>Temperature drift</td>
<td>Switch on for 0 or 2 more hours</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2 Results of the $\lambda$-cut matrix $R_\lambda$

| Condition                  | Minimum value $\min(R_\lambda)$ | Total no. of zero value elements $N(R_\lambda|i,j=0)$ | Minimum value $\min(R_\lambda)$ | Total no. of zero value elements $N(R_\lambda|i,j=0)$ |
|----------------------------|--------------------------------|-----------------------------------------------------|--------------------------------|-----------------------------------------------------|
| PSI VSI                    |                                |                                                     |                                |                                                     |
| No change                  | 1                              | 0                                                   | 1                              | 0                                                   |
| Change of magnification    | 0                              | 24                                                  | 0                              | 16                                                  |
| Change of location         | 0                              | 25                                                  | 0                              | 16                                                  |
| Focusing                   | 0                              | 10                                                  | 1                              | 0                                                   |
| Warm up                    | 0                              | 22                                                  | 0                              | 22                                                  |
| Temperature drift          | 0                              | 22                                                  | 1                              | 0                                                   |

Conclusions

Experiments conducted on the optical profiler are presented. Based on the analysis of the experimental results, the proposed method has shown to be quite effective in identifying the systematic errors. The method has shown to be effective for small changes, as long as the measurement resolution is sufficient, in the system condition that could be difficult to detect.

References