Evaluation of Measurement Uncertainty in Gear Measuring Instruments by Using the Monte Carlo Simulation

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Introduction

Recently, the need to perform uncertainty estimation is increasing, and many examples of uncertainty estimation have been presented. However, most of the examples can be expressed by model equations, and very few examples of complex quantities (i.e. industrial quantities) have been given. Because the factors involved in uncertainties are intricately interconnected in industrial quantities, it is very difficult to calculate the sensitivity coefficients of these factors.

The same is true also for gear measurement. In Japan, the Japan Gear Manufacturers Association established a committee of gear measurement accuracy for investigating gear measurement traceability and calibrations. One of the purposes of the committee is to estimate the uncertainty of gear shape measurement.

The geometry of a gear measurement machine, which is an industrial quantity measurement machine, is complex, and it is very difficult to estimate the uncertainty by applying the law of propagation of uncertainty which is defined by the Guide to the Expression of Uncertainty in Measurement (GUM). In this study, we estimate the geometrical uncertainties of gear measurement by using the Monte-Carlo simulation.

Gear measurement machine

The architecture of a gear measurement machine is shown in Fig.1. A linear slide is parallel with the y (tangent)-axis. A probe is attached to the linear slide. When the gear shape is measured, the probe is located at a point in the direction of the x (radial)-axis distant from the origin by the radius of a base circle. If the gear is turned, the tooth of the gear pushes the probe, and the length of probe movement is measured as y. There is a linear relation between the length of probe movement y and the angle of rotation \( \theta \).

\[ y = r \theta \]

Here, \( r \) means radius of base circle. The difference between the measured value and ideal value is the profile deviation.

Sources of geometrical uncertainties in gear measurement

Because the geometry of gear measurement machine is complex, it is very difficult to estimate total geometrical uncertainty. However, it is possible to estimate individual geometrical uncertainties. The sources of individual uncertainties which are considered in this study are shown in Fig.2 and Fig.3.

Fig.1: The architecture of gear measurement machine

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2 The University of Electro-Communications
3 Osaka Seimitsu Kikai Co., Ltd.
4 Kanzaki Kokyukoki Mfg. Co., Ltd.
5 Tokyo Technical Instruments Inc.
6 Asano Gear Co., Ltd.
7 Ogasawara Precision Hob Laboratory Ltd.
8 National Institute of Advanced Industrial Science and Technology
The preceding individual uncertainties were measured at the University of Electro-Communications and Osaka Seimitsu Kikai Co., Ltd\(^1\). The results of the measurements are shown in Table\(^1\).\(^2\) These results were substituted by a Monte-Carlo simulator, and we thereby obtained an uncertainty of gear tooth measurement.

### Monte-Carlo simulation and result

Measurement and simulation conditions are as defined below,

- Rotation angle of gear: \(37\text{deg.}\)
- Number of partitions of rotation angle: \(45\)
- Radius of base circle: \(49.4\text{mm}\)
- Model of tooth: \(300,000\) points per line, \(0.1\text{mm}\) pitch in the \(z\)-axis direction
- Repetitions: \(1000\)

#### Table1: Individual uncertainties

<table>
<thead>
<tr>
<th>Source of uncertainties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eccentricity ((x)-axis direction)</td>
<td>(0.51\mu m)</td>
</tr>
<tr>
<td>Eccentricity ((y)-axis direction)</td>
<td>(0.51\mu m)</td>
</tr>
<tr>
<td>Origin setting error ((x)-axis direction)</td>
<td>(0.32\mu m)</td>
</tr>
<tr>
<td>Origin setting error ((y)-axis direction)</td>
<td>(0.12\mu m)</td>
</tr>
<tr>
<td>Length measurement uncertainty ((x)-axis direction)</td>
<td>(0.3\mu m)</td>
</tr>
<tr>
<td>Length measurement uncertainty ((y)-axis direction)</td>
<td>(0.0296\mu m)</td>
</tr>
<tr>
<td>Length measurement uncertainty ((z)-axis direction)</td>
<td>(0.3\mu m)</td>
</tr>
<tr>
<td>Pitching of (y)-axis</td>
<td>(3.73\mu m)</td>
</tr>
<tr>
<td>Yawing of (y)-axis</td>
<td>(0.684\mu m)</td>
</tr>
<tr>
<td>Angle of (x)-axis for the reference plane</td>
<td>(0.001\mu m/100\text{mm})</td>
</tr>
<tr>
<td>Angle of (x)-axis against the reference plane</td>
<td>(0.001\mu m/100\text{mm})</td>
</tr>
<tr>
<td>Angle of (y)-axis for the reference plane</td>
<td>(0.000245\text{rad})</td>
</tr>
<tr>
<td>Angle of (y)-axis against the reference plane</td>
<td>(0.000118\text{rad})</td>
</tr>
<tr>
<td>Uncertainty of an angle of rotation</td>
<td>(0.00000242\text{rad})</td>
</tr>
<tr>
<td>Slope of axel ((x)-axis direction)</td>
<td>(0.0000060\text{rad})</td>
</tr>
<tr>
<td>Slope of axel ((y)-axis direction)</td>
<td>(0.000154\text{rad})</td>
</tr>
</tbody>
</table>

The preceding individual uncertainties were measured at the University of Electro-Communications and Osaka Seimitsu Kikai Co., Ltd\(^1\). The results of the measurements are shown in Table\(^1\).\(^2\) These results were substituted by a Monte-Carlo simulator, and we thereby obtained an uncertainty of gear tooth measurement.

**Monte-Carlo simulation and result**

Measurement and simulation conditions are as defined below,

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- Repetitions: \(1000\)
Individual uncertainties shown in Table 1 are limited values except the length measurement uncertainty (y-axis direction). The distributions of individual uncertainties except the length measurement uncertainty (y-axis direction) are assumed to be rectangular distributions, and the distribution of the length measurement uncertainty (y-axis direction) is assumed to be a normal distribution. The detail of the simulation is given below. First, a virtual machine of the gear measurement machine is programmed. This virtual machine has factors of geometrical uncertainties which are the same as the factors of a real gear measurement machine, and random values which conform to the fixed distributions are generated. The random values are inputted into the virtual machine, and we obtain an output which represents the measurement results including whole geometrical uncertainties. If this procedure is repeated, we obtain the distribution of measurement results, and we can calculate expanded uncertainties. The detail of the estimation uncertainty flowchart is shown in Fig. 4. Here, the number of repetition is 1000 times, and the results are arranged by size starting with the smallest. The 950th value from this size represents expanded uncertainty as the expanded uncertainty is an interval having a level of confidence of approximately 95%.

![Flowchart of the simulation](image-url)
The results of the simulation are shown below. We estimate three uncertainties of profile deviations which are prescribed by ISO 1328-1:1995, namely total profile deviations, profile form deviations and profile slope deviations. The results of estimating uncertainty are given below:

- Measurement uncertainty of total profile deviations: 0.73µm
- Measurement uncertainty of profile form deviations: 0.43µm
- Measurement uncertainty of profile slope deviations: 0.49µm

One of the results which was obtained from 1000 simulations is shown in Fig. 5, the histogram of profile form deviation is shown in Fig. 6, the histogram of profile slope deviation is shown in Fig. 7 and the histogram of total profile deviation is shown in Fig. 8.

The histogram of the profile form deviation is a normal distribution, but the histogram of the profile slope distribution decreases linearly. The reason is that the profile form deviation is a random error, and the profile slope deviation is a systematic error. The total profile deviation is a combined profile form deviation with profile slope deviation, which is an asymmetrical distribution. Generally, in an asymmetrical distribution, it is difficult to calculate expanded uncertainty, but in this case, it is very easy to find the 950th value which represents expanded uncertainty.

Conclusions
This study estimates the expanded uncertainty of a measuring gear tooth profile which is a very complex value. However, there is the possibility of overestimation in order to apply the distribution of a limited value to a rectangular distribution. This point must be improved. In the future, we will structure the simulation for helix deviations and pitch deviations as well as profile deviations.

Bibliography