The Application of an Empirical Tool Force Model on Vibration Assisted Cutting

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Introduction

Many experiments have demonstrated that vibration assistance can make machining forces decrease. Several researchers have presented force models for single point cutting with vibration. Astashev presents a model for cutting with tool vibration parallel to the cutting direction [1]. Dow et al. describe a force model for elliptical cutting [2]. In this paper we develop a force model for cutting with vibration in the thrust direction. We modify an empirical tool force model incorporating elastic and plastic material properties that was developed for diamond turning [3].

Figure 1 shows a 2-dimensional view of the orthogonal cutting process. Key parameters include depth of cut ($D$), cutting speed ($V$), tool tip radius ($r$), shear angle ($\phi_s$), clearance angle ($\alpha$), spring back ($\gamma_{rb}$) and rake angle ($\gamma_{ra}$). Some of these parameters are prescribed: depth of cut, cutting speed, clearance angle and rake angle. The figure also shows three sets of forces: shear plane ($F_{sf}$ and $F_{sn}$), tool edge ($F_{ex}$ and $F_{ey}$) and clearance face ($F_{cf}$ and $F_{cn}$). The shear plane forces can be represented as rake face forces with a coordinate transformation. Tool forces are thus divided into three zones: rake face, tool edge, and clearance face.

Introducing high frequency vibration in the vertical direction (applied to tool or workpiece) causes most parameters to change during the machining process. These changes affect the forces on the three tool zones. For example, the relative cutting speed between tool and workpiece changes in amplitude and direction. This results in the possibility of a friction force direction
change on the rake face. Also, vibration changes the contact length between the tool edge and workpiece so that the cutting and thrust force on the tool edge area differ from the forces without vibration. Meanwhile, the contact length between the clearance face and workpiece changes, as does the effective clearance angle. This makes the normal force and friction force on this cutting zone different from the situation without vibration. The machining forces on these three zones can be investigated separately as described below.

**Forces on the Rake Face**

Some assumptions need to be made in order to calculate the normal force and friction force on the rake face [3]. First, the workpiece material is regarded as elastic-perfectly plastic. Second, the normal stress on the shear zone is the uni-axial material flow stress, i.e. $\sigma_s = H/3$. Third, Von Mises failure criterion is employed so that shear stress is

$$\tau_s = \sigma_s / \sqrt{3} = H / 3\sqrt{3}$$  \hspace{1cm} (1)

Finally, the shear plane angle with respect to a fixed coordinate system remains constant and consequently the shear angle with respect to the cutting direction varies [4]. The normal force and shear force on the shear plane should be:

$$F_{sn} = \sigma_s A_s = \frac{H}{3} \frac{A_c}{\sin \phi_s} = \frac{HL}{3 \sin \phi_s} (D - a \sin \omega \phi)$$  \hspace{1cm} (2)

$$F_{sf} = \tau_s A_s = \frac{HL}{3 \sin \phi_s} \frac{D - a \sin \omega \phi}{\sqrt{3}}$$  \hspace{1cm} (3)

where $A_c$ = uncut chip area, $L$ = tool/chip contact length projected onto surface of workpiece, $A_s$ = shear plane area, $a$ = vibration amplitude and $\omega$ = vibration angular velocity.

With respect to the rake face, these forces can be rewritten as[5][6]:

$$F_{rf} = F_{sn} \cos(\phi_s - \gamma_{ne}) - F_{sf} \sin(\phi_s - \gamma_{ne})$$  \hspace{1cm} (4)

$$F_n = F_{sn} \sin(\phi_s - \gamma_{ne}) + F_{sf} \cos(\phi_s - \gamma_{ne})$$  \hspace{1cm} (5)

where $F_{rf}$ = friction force on the rake face, and $F_n$ = normal force on the rake face.

The direction of the rake face friction force depends on the relative velocity between the chip and tool. Under certain conditions, it is possible for the vibration velocity ($V_a$) to exceed the chip velocity ($V_c$). At these times, the friction force between the chip and tool changes direction. Consequently, when averaged over many cycles, the friction force is less than its nominal value.

Figure 2 shows an example of a case where the friction force points up the rake face between $t_0$ and $t_1$, down the rake face between $t_1$ and $t_2$, and back up the rake face between $t_2$ and $t_3$. 
In these three regions, the time weighted friction force is:

\[
F_{rfv1} = \frac{HL}{3\sin \phi_s} \left[ \int_{t_0}^{t_1} (D - a \sin \alpha) \cos(\phi_s - \gamma_ne) - \frac{1}{\sqrt{3}} (D - a \sin \alpha) \sin(\phi_s - \gamma_ne) dt \right]
\]

\[
F_{rfv2} = \frac{HL}{3\sin \phi_s} \left[ \int_{t_1}^{t_2} (D - a \sin \alpha) \cos(\phi_s - \gamma_ne) - \frac{1}{\sqrt{3}} (D - a \sin \alpha) \sin(\phi_s - \gamma_ne) dt \right]
\]

\[
F_{rfv3} = \frac{HL}{3\sin \phi_s} \left[ \int_{t_2}^{t_3} (D - a \sin \alpha) \cos(\phi_s - \gamma_ne) - \frac{1}{\sqrt{3}} (D - a \sin \alpha) \sin(\phi_s - \gamma_ne) dt \right]
\]

We define “time weighted force” as \(F_{\text{weighted}} = F_{\text{actual}} \Delta t/T\) where \(\Delta t\) is the time over which the force acts and \(T\) is the vibration period. The average friction force on the rake face is

\[F_{rf} = F_{rfv1} - F_{rfv2} + F_{rfv3}\]

Without vibration, the friction force on the rake face is:

\[
F_f = \frac{HL}{3\sin \phi_s} \left[ D \cos(\phi_s - \gamma_ne) - \frac{1}{\sqrt{3}} \sin(\phi_s - \gamma_ne) \right]
\]

To illustrate the effect of vibration on the rake face forces, we consider the following cutting condition: \(V=1\text{m/s}, \gamma_ne=5^\circ, f=5000\text{Hz}, a=12\mu\text{m}, D=20\mu\text{m}, \phi_s=30^\circ, \alpha=8^\circ, r=5\mu\text{m}, H=5.3\text{GPa},\) and \(L=0.01\text{m}.\) In this case, \(F_{rfv1}=104.25\text{N}, F_{rfv2}=34.5\text{N},\) and \(F_{rfv3}=41.45\text{N}.\) The friction force on the rake face with vibration is therefore \(F_{rfv}=112\text{N}.\) The friction force without vibration \(F_f=180.2\text{N}\) However, the normal force keeps constant with vibration, i.e. \(F_{rn}=F_{rn}=397.84\text{N}\)

**Force on the Tool Edge**

Figure 3 shows the interface stress \((\sigma_t)\) acting on the tool edge. The forces \(F_{ex}\) and \(F_{ey}\) can be computed based on \(\sigma_t,\) the edge contact length \(l_e,\) and the tool width (into the paper) \(L.\)
An empirical formula for the tool/workpiece interface stress $\sigma_f$ is given by [3]:

$$\sigma_f = K_2 H \sqrt{\frac{H}{E}} \quad \text{where } K_2 \text{ is a constant} \quad (10)$$

The endpoint of $l_e$ is tangential to the resultant tool velocity. Without vibration, the contact length between the tool edge and workpiece is

$$l_e = \left(\frac{\pi}{2} + \gamma_{ne}\right) r \quad (11)$$

The total force in the horizontal and vertical directions are given by:

$$F_{ex} = \sigma_f L l_e \left[ \sin \left( \frac{\pi}{4} + \frac{\gamma_{ne}}{2} \right) + \mu \cos \left( \frac{\pi}{4} + \frac{\gamma_{ne}}{2} \right) \right] \quad (12)$$

$$F_{ey} = \sigma_f L l_e \left[ \cos \left( \frac{\pi}{4} + \frac{\gamma_{ne}}{2} \right) - \mu \sin \left( \frac{\pi}{4} + \frac{\gamma_{ne}}{2} \right) \right] \quad (13)$$

With vibration: $l_e = \left(\frac{\pi}{2} + \gamma_{ne} + \alpha_s\right) r \quad (14)$

where $\alpha_s$ is defined as an angle between the tangential point of resultant cutting speed and the beginning of the straight flank area. Under the same conditions mentioned previously and with $\mu = 0.9$, the maximal absolute value $|\alpha_s| = \text{atan}(\omega_{a}/V) = 20.66^\circ$

We define the time weighted forces $F_{ex1}$ and $F_{ex2}$ as follows. In the half vibration cycle, from $t_0 = 0$ to $t_1 = 6.2158 \times 10^{-5} \text{sec}$, $\alpha_s$ increases from $-20.66^\circ$ to $8^\circ$,

$$F_{ex1} = \frac{2}{T} \sigma_f L l_e \left[ \sin \left( \frac{\pi}{4} + \frac{\beta}{2} - \frac{a \tan \frac{\alpha_s \cos \alpha}{V}}{2} \right) + \mu \cos \left( \frac{\pi}{4} + \frac{\beta}{2} - \frac{a \tan \frac{\alpha_s \cos \alpha}{V}}{2} \right) \right] dt \quad (15)$$

Then $\alpha_s$ remains at $8^\circ$ until $t_2 = 1 \times 10^{-4} \text{sec}$ (end of the half vibration cycle) due to the clearance angle limitation. The limitation is because the resultant speed could vary from $-20.66^\circ$ to $20.66^\circ$. However, because the clearance angle is smaller than $20.66^\circ$, the variation of $\alpha_s$ is from $-20.66$ to $8^\circ$. 

![Fig.3: Normal stress distribution on the finished surface](image-url)
For the second half of the vibration cycle:

\[
F_{e_{x2}} = \frac{2}{T} \sigma_f L \left( \frac{\pi}{2} + \gamma_{ne} + \alpha \right) r(t_2 - t_1) \left[ \sin \left( \frac{\pi}{4} + \frac{\gamma_{ne}}{2} - \frac{\alpha}{2} \right) + \mu \cos \left( \frac{\pi}{4} + \frac{\gamma_{ne}}{2} - \frac{\alpha}{2} \right) \right]
\]  

(16)

So the cutting force is \( F_{ex} = F_{e_{x1}} + F_{e_{x2}} \). Introducing a value for interface stress \( \sigma_f = 3.54 \text{ GPa} \)

The cutting and thrust force without vibration are \( F_{ex} = 394.82 \text{ N} \) and \( F_{ey} = 3.52 \text{ N} \). With vibration, \( F_{e_{x1}} = 222.3 \text{ N} \), \( F_{e_{x2}} = 161.49 \text{ N} \), and \( F_{e_{xv}} = 383.79 \text{ N} \). Because \( F_{ey} \) is very small compared to \( F_{ex} \), it was not calculated.

**Force on the Clearance Face**

The analysis on the clearance face includes two parts: the straight flank area and a small arc where the direction of normal stress \( \sigma_f \) is not constant. In Figures 4 and 5, the left starting point of the small arc is defined by the tangential point of the resultant cutting speed and the right ending point of the small arc is the beginning of the straight flank area. Suppose spring back \( s \) is 6 \( \mu \text{m} \), the machining forces on the clearance face without vibration are:

\[
F_{cx} = \sigma_f L r \alpha \left[ \sin \left( \frac{\alpha}{2} \right) - \mu \cos \left( \frac{\alpha}{2} \right) \right] + \sigma_f L \left[ s - r(1 - \cos \alpha) \right] \sin \alpha - \mu \cos \alpha \]  

(17)

\[
F_{cy} = \sigma_f L r \alpha \left[ \cos \left( \frac{\alpha}{2} \right) + \mu \sin \left( \frac{\alpha}{2} \right) \right] + \sigma_f L \left[ s - r(1 - \cos \alpha) \right] \cos \alpha + \mu \sin \alpha \]  

(18)

With vibration, we determine the time weighted forces \( F_{cvl} \) and \( F_{cv2} \). In the half vibration cycle, from \( t_0 = 0 \) to \( t_1 = 6.2158 \times 10^{-5} \text{ sec} \), \( \alpha_s \) increases from \(-20.66^\circ\) to \(8^\circ\), and

\[
F_{cvl} = \int_{t_0}^{t_1} (\alpha + \alpha_s) r \left[ \sin \left( \frac{\alpha_s}{2} - \frac{\alpha}{2} \right) + \mu \cos \left( \frac{\alpha_s}{2} - \frac{\alpha}{2} \right) \right] dt
\]

\[
F_{cv2} = \int_{t_0}^{t_1} (\alpha + a \alpha_s) r \left[ \cos \left( \frac{\alpha_s}{2} - \frac{\alpha}{2} \right) - \mu \sin \left( \frac{\alpha_s}{2} - \frac{\alpha}{2} \right) \right] dt
\]

(19)

(20)

Then \( \alpha_s \) remains at \(8^\circ\) until \( t_2 = 1 \times 10^{-4} \text{ sec} \), and
\[
F_{cxv2} = \int_0^\infty [s - r(1 - \cos \alpha)](\sin \alpha - \mu \cos \alpha)dt
\]

\[
F_{cyv2} = \int_0^\infty [s - r(1 - \cos \alpha)](\cos \alpha + \mu \sin \alpha)dt
\]

So, the cutting force is \( F_{cx} = F_{cxv1} + F_{cxv2} \), and the thrust force is \( F_{cy} = F_{cyv1} + F_{cyv2} \). Using the same conditions described previously, the cutting and thrust forces without vibration are \( F_{cx} = 184.59 \) N and \( F_{cy} = 222.92 \) N. With vibration, \( F_{cxv1} = 24.72 \) N, \( F_{cxv2} = 171.37 \) N and \( F_{cxv} = 196.09 \) N. And \( F_{cyv1} = 25.35 \) N, \( F_{cyv2} = 227.53 \) N and \( F_{cyv} = 252.88 \) N.

**Combined Force**

Summing the forces from the three zones, the cutting and thrust forces without vibration are:

\[
F_x = F_{rf} \sin \gamma_{ne} + F_{rn} \cos \gamma_{ne} + F_{ex} + F_{cx} = 991.44N
\]

\[
F_y = F_{rf} \cos \gamma_{ne} - F_{rn} \sin \gamma_{ne} + F_{ey} + F_{cy} = 371.28N
\]

With vibration, the forces would be:

\[
F_{xv} = F_{rfv} \sin \gamma_{ne} + F_{rnv} \cos \gamma_{ne} + F_{exv} + F_{cxv} = 985.97N
\]

\[
F_{yv} = F_{rfy} \cos \gamma_{ne} - F_{rny} \sin \gamma_{ne} + F_{eyv} + F_{cyv} = 333.3N
\]

For this case, the cutting force decreases by 0.55\%, and thrust force decreases by 10.2\%.

**Conclusions**

Using an existing empirical tool force model, together with a simple geometric assumption about the shear angle, we can predict that moderately high vibration frequencies (thousands of Hertz) can produce significant force reductions, especially on the thrust force. Experiments will be done to validate the theoretical results. Future work will also investigate the effect of vibration parameters and tool geometry on the total force reduction.

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**References**


