NanoSlider Modeling and Waveform Adaptation for Positioning Control

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Introduction
Ultraprecision positioning mechanism with both long moving range and nanometer resolution is more and more required in the field of precision engineering (STM, AFM) and many efforts are made to meet these requirements such as inertial slider, inchworm etc. We have reported a nanopositioner which has long moving range and high resolution at the same time [1][2]. It uses inertial sliding principle and can move in the three-planar direction(XY0).
In the previous report only the experimental movement characteristics were presented. In this paper the dynamic modeling of the nanoslider and with that result input waveform to the piezoelectric actuator are discussed. With the assumption that the dynamic characteristic of the piezoelectric tube can be explained by the simple mass-spring-damper model the dynamic model of the nanoslider can be obtained. Dynamic equation formulation will be performed and its validity will be verified with the experiment. The experimental setup includes the three-axis laser interferometer and mirrors mounted to the body for reflecting laser beam(Fig. 1). With the model waveform shape of the input voltage can be determined to improve the performance. The considerations taken for waveform are the oscillation at the end of the actuation step and speed of the movement.

Fig. 1. Nanoslider and 3-axis interferometer view

Experimental investigation and friction uncertainty region
Many experiments with sawtooth input shows that under certain conditions there are significant friction condition uncertainties. The changes of the position of the nanoslider, moving path history and the motion direction(x and y) make the responses quite different and the resultant displacement is varied. This uncertain region is characterized by the input voltage and frequency : below certain voltage/about 2μm and above certain frequency(200Hz). By the
small voltage input, the microscopic structure of the contacting surfaces becomes dominant and by the high frequency input (above natural frequency of the nanoslider in the motion direction) there exist so many sliding-conditions occurring and with the surface character generated by the sliding effect make the responses different (worst case is 100% error). However if the voltage is high enough and the frequency is low enough, the uncertainties become much lower (about 5%) and shows repeatable response characteristics. Therefore the modeling and simulation are based on the responses in this region and positioning control will be operated also.

**Modeling and experimental results**

The modeling of the nanoslider includes both the stick phase model and sliding phase model. The stick phase model will be used in the control when stick and theoretical aspects can assist in the controlling nanoslider such as gain tuning and control method (stick-control mode).

![Diagram](image)

**Fig. 2. Assumptions of the model**

With some reasonable assumptions on the piezoelectric tube (Fig. 2), we can simplify the total nanoslider mechanism to a mass-spring-damper model as shown in Fig. 3. Large mass M is nanoslider, small mass m is tube equivalent mass (of which relative position is tube end relative position), k is tube equivalent stiffness and c is the damping coefficient. Fig. 3 shows the coordinate systems and schematic of the various components of the model. To make the dynamic equation of motion Lagrange's equation is used and first the sliding equation is formulated.

\[
T = \frac{1}{2} M \dot{N} v_{M,0}^2 + \frac{1}{2} I \dot{N} w_{0}^2 + \frac{1}{2} \sum_{i=1}^{p} m_i \dot{N} v_{p_i}^2
\]

\[
V = \frac{1}{2} \sum_{j=1}^{p} k_j ( \dot{N} r_j - \dot{N} r_{ij} ) \cdot ( \dot{N} r_j - \dot{N} r_{ij} )
\]

\[
= \frac{1}{2} \sum_{j=1}^{p} k_{ij} ( \dot{y}_{ji}^2 + \dot{y}_{ji}^2 )
\]

(1)
\[
D = \frac{1}{2} \sum_{k=1}^{n} c_k \left( \dot{x}_{ph} \cdot \dot{x}_{ph} \right) \\
= \frac{1}{2} \sum_{j=1}^{L} c_j \left( x_{ph}^2 + y_{ph}^2 \right)
\]

Fig. 3. Nanoslider model: m is mass, k is spring constant, c is damping of the tube piezoelectric material. Right hand figure shows the external and internal forces and coordinates used for model.

With the above equation and some labor, mass, spring and damping matrix can be obtained and shown below.

\[
M = \\
\begin{bmatrix}
M + \sum m_i, & 0, & \sum m_i \xi y_i & m_{13} & 0 & m_{21} & 0 & m_{31} & 0 \\
0, & M + \sum m_i, & \sum m_i \xi y_i & m_{13} & 0 & m_{21} & 0 & m_{31} & 0 \\
-\sum m_i \xi y_i & \sum m_i \xi y_i & M + \sum m_i (\xi y_i + \xi y_i) & -m_{13} & -m_{13} & m_{13} & 0 & m_{13} & 0 \\
0, & m_{13}, & m_{13}, & 0, & m_{13}, & 0, & 0, & 0, & 0 \\
0, & m_{23}, & m_{23}, & 0, & m_{23}, & 0, & 0, & 0, & 0 \\
0, & m_{33}, & m_{33}, & 0, & m_{33}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & m_{13}, & 0, & 0, & 0, & m_{13}, & 0 \\
0, & 0, & 0, & 0, & m_{23}, & 0, & 0, & 0, & m_{23} \\
0, & 0, & 0, & 0, & 0, & m_{33}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & m_{33}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & m_{33}, & 0 \\
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & k_1, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & k_2, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & k_2, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & k_3, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & k_3 \\
\end{bmatrix} \\
C = \begin{bmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
\end{bmatrix}
\]
\[
    f = \begin{bmatrix}
    \sum F_{fx} \\
    \sum F_{fy} \\
    \sum F_{gz} - F_{kz} - F_{kx} - F_{ky} + F_{kz} - F_{kx} - F_{ky}
    \end{bmatrix}
\]

And the total sliding equation is,

\[
    M \ddot{q} + C \dot{q} + Kq = f
\]  

(3)

When the contacting ball and surface is stick this equation holds : which says the absolute velocity of the ball end point is zero.

\[
    \begin{cases}
    \dot{q}_1 = \frac{G_{y_1}}{G_{x_1}} \dot{q}_3 + \frac{G_{x_2}}{G_{x_1}} \\
    \dot{q}_2 = \frac{G_{x_1}}{G_{x_2}} \dot{q}_3 + \frac{G_{y_2}}{G_{x_1}} = N \frac{v_{x_2}}{x_2} = 0 \\
    \end{cases}
\]

(4)

And the stick equation is also derived from eq. (3) and (4). The dynamic model includes sliding and stick model and it is switched to each other, therefore the model is nonlinear. To simulate the response of the nanoslider the model equation is solved numerically using adaptive Runge-Kutta method. The parameters such as mass and damping are measured using FFT and step-responses and sinusoidal inputs. And the sliding friction coefficient is determined from the simulation comparing with the experimental result and error minimized method determines the value.

Figure 5 shows the compared results with the experimental and simulation results. The input shape is sawtooth and the voltage is 80 V (total 160 V considering two opposite electrodes). The input frequency is varied from 100Hz to 200Hz. (a) and (b) is the x and y results repectively and (c) is the simulation results in the y-direction. The response characteristics and final displacement value are almost the same with respect to the tendency according to the input frequency. Thus the dynamics of the model can simulate the nanoslider adequately.

**Waveform determination**

With the model waveform shape of the input voltage can be determined to improve the performance. The considerations taken for waveform are the oscillation at the end of the actuation step and speed of the movement. The waveform is sectioned by the four segments and each segment is polynomial function or spline curve and the function parameter is varied and the movement performance is estimated numerically with the model. Therefore suitable waveform can be determined from the simulation process. However the nonlinearity and uncertainty exists
in the friction phenomena can make the real situation somewhat different from the simulation result, therefore the adaptation in the waveform should be made experimentally when applied to the real system.

![Graphs](image)

(a)  (b)  (c)

Fig. 5. Comparison between simulation and experiment: (a) and (b) are experiments and (c) is simulation

**Conclusions**

Through many experimental investigations, the uncertain region and repeatable region are known. And the modeling based on the repeatable region is performed and its validity is verified by comparison with the real phenomena. The further work will be launched by the waveform determination and follows the closed loop control of the nanoslider.

**References**


