INTRODUCTION

Variable period scanning beam interference lithography (VP-SBIL) [1], [2] is under development in our laboratory. It is a generalized concept of phase-locked scanning beam interference lithography [3], [4], [5] for patterning continuously varying (chirped or quasi-periodic) gratings. The detailed description of the design and operation of the VP-SBIL can be found in Ref. [2].

For sub-nanometer phase distortions across the substrate, the VP-SBIL calls for nanometer-accurate fringe metrology immune to variations in the beam angle and orientation since it enables the precise stitching of adjacent scans [5]. Previously, a novel interference fringe metrology has been proposed, which exploits two unique properties of Fresnel zone plates: (1) they contain a wide range of spatial frequencies, and (2) all spatial frequencies are phase symmetric with respect to the optical axis [6]. For the system, it is highly important to investigate the signal contrast variations with changing incident angles since the accurate measurement of fringe characteristics is critically dependent on the determination of signal properties, i.e., peaks and valleys.

In this paper, we report the theoretical study and simulation results on the contrast variations with changing incident angles.

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THEORETICAL MODEL AND SIMULATION RESULTS

Consider two plane waves incident upon a Fresnel zone plate (Figure 1). The irradiance distribution in the far-field plane,\((x, y)\), which is at a distance\(z\) from the Fresnel zone plate, is given by

\[
I(x, y) = g_1(x, y)^2 + g_2(x, y)^2 + 2g_1(x, y)g_2(x, y)\cos \phi
\]

where \(g_1(x, y)\) and \(g_2(x, y)\) are the far-field diffraction patterns of the Fresnel zone plate corresponding to each plane wave (FFZP’s), and \(\phi\) is the phase difference between the two plane waves. Here, \(2g_1(x, y)g_2(x, y)\) represents the far-field Moiré zone plates (FMZP) described in Ref. [6]. If two plane waves are assumed to have the same amplitude and incident angle \(\theta\), \(g_1(x, y)\) and \(g_2(x, y)\) have the same amplitude functions except that they are mirror symmetric about the \(z\) axis. Then, (1) can be re-written as,

\[
I(x, y) = g(x + s, y)^2 + g(x - s, y)^2 + 2g(x + s, y)g(x - s, y)\cos \phi
\]

where \(s\), given by \(s = z\sin \theta\), is the distance between the origin and the center of the FFZP, and \(g(x, y)\) is a circularly symmetric chirped function.

If the irradiance (2) is integrated over the innermost zone of the first FMZP centered at the origin, the power is obtained as

\[
P(s) = \int \int_A \left\{ g(x + s, y)^2 + g(x - s, y)^2 \right\} dA + 2 \int \int_A g(x + s, y)g(x - s, y) dA \cos \phi = D_A(s) + N_A(s) \cos \phi,
\]

and a contrast function can then be defined as,

\[
C_A(s) = \left| \frac{\int \int_A 2g(x - s, y)g(x + s, y) dA}{\int \int_A \{g(x - s, y)^2 + g(x + s, y)^2\} dA} \right| = \frac{|N_A(s)|}{D_A(s)}.
\]

The contrast would be unity, if \(g(x - s, y) = \pm g(x + s, y)\) over the innermost zone of the first FMZP. However, since \(g(x, y)\) is a quadratic chirped function, \(g(x - s, y)\) cannot be equal to \(\pm g(x + s, y)\) over the region for \(s \neq 0\), meaning that the contrast cannot be unity.

To find the values of \(s\) where the contrast takes on maximum or minimum values, let us assume that the distance \(s\) between the origin and the center of FFZP is large enough that the FMZP can be produced. If \(s\) is not large enough, the FMZPs would no longer be formed, instead linear fringes will appear [7].

The defined integral region covers many outer zones of the FFZP. It is intuitive that \(D_A(s)\) is not much affected by the change of \(s\). The in-and-out movements of fine outer zones in the integral region do not greatly affect the values of \(D_A(s)\) over the range of \(s\) considered, so that \(\frac{dD_A(s)/ds}{D_A(s)} \ll 1\). (5)

For the values of \(s\) at which the contrast has extrema, it must be true that

\[
\frac{dC_A(s)}{ds} = \left| \frac{dN_A(s)}{ds}D_A(s) - N_A(s)\frac{dD_A(s)}{ds}}{D_A(s)^2} \right| = 0.
\]

Noting that the second term of the numerator in Equation (6) is negligible, and \(D_A(s)\) cannot be zero in the innermost zone of the first FMZP, \(dN_A(s)/ds = 0\) is obtained. If we assume that there is no phase difference between the two plane waves (\(\phi = 0\)), the derivative of power is expressed as,

\[
\frac{dP(s)}{ds} = \frac{dN_A(s)}{ds} + \frac{dD_A(s)}{ds} = 0
\]

Hence, the power has its extrema at the values of \(s\) where the contrast has its extrema. Furthermore, the maxima in contrast are observed when the power has extreme values over the integral region. The power reaches the peak or valley values when \(N_A(s)\) has the maximum values in magnitude, but the opposite signs
Figure 2: Simulated contrast variations at different incident angles. An amplitude Fresnel zone plate with an innermost zone radius of 265 µm is used. (a) incident angle = 17.19°, contrast = 0.59, (b) incident angle = 17.58°, contrast = 0.25, (a) incident angle = 18.06°, contrast = 0.54.
in each case. Figure 2 demonstrates the contrast values and the signals produced by the phase difference at different incident angles. Note that the maxima in contrast are obtained when the power over the innermost zone of the first FMZP reaches its extrema as mentioned before.

Now let us assume the amplitude function \( g(x, y) \) as shown below,

\[
g(x, y) = \sum_{n=-\infty}^{\infty} A_n \exp[ik_n(x^2 + y^2)]
\]  

(7)

where \( A_n = A_{-n}^* \) since \( g(x, y) \) is real. Then,

\[
g(x + s, y)g(x - s, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} A_m A_n \exp[ik_m((x + s)^2 + y^2)] \exp[ik_n((x - s)^2 + y^2)]
\]

\[
= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} A_mA_n \exp[i(k_m + k_n)(x^2 + 2sx + k_m - k_n)x + y^2 + s^2]].
\]

(8)

Since the first FMZP centered at the origin is being considered, \( m = n \) [7]. Therefore,

\[
g(x + s, y)g(x - s, y) = \sum_{n=-\infty}^{\infty} A_n^2 \exp[i2k_n(x^2 + y^2 + s^2)],
\]

(9)

and

\[
N(s) = \int \int_A 2g(x + s, y)g(x - s, y)dA
\]

\[
= \int \int_A 2 \sum_{n=-\infty}^{\infty} A_n^2 \exp[i2k_n(x^2 + y^2 + s^2)]dA
\]

\[
= \sum_{n=-\infty}^{\infty} 2A_n^2 \int \int_A \exp[i2k_n(x^2 + y^2 + s^2)]dA
\]

\[
= \sum_{n=-\infty}^{\infty} 2A_n^2 \exp(i2k_n s^2) \int \int_A \exp[i2k_n(x^2 + y^2)]dA
\]

\[
= \sum_{n=-\infty}^{\infty} 2A_n^2 B_n \exp(i2k_n s^2),
\]

(10)

where \( B_n = \int \int_A \exp[i2k_n(x^2 + y^2)]dA \). The contrast function has zeros at the values of \( s \) where \( N_A(s) \) satisfies

\[
N(s) = \sum_{n=-\infty}^{\infty} 2A_n^2 B_n \exp(i2k_n s^2) = 0.
\]

(11)

The contrast also has maximum values if

\[
\frac{dN_A(s)}{ds} = \frac{d \left[ \sum_{n=-\infty}^{\infty} 2A_n^2 B_n \exp(i2k_n s^2) \right]}{ds}
\]

\[
= 8si \sum_{n=-\infty}^{\infty} C_n \exp(i2k_n s^2) = 0
\]

(12)
where \( C_n = k_n A_n^2 B_n \). Therefore, the contrast has the maxima at the values of \( s \) which satisfy the following equation

\[
\sum_{n=-\infty}^{\infty} C_n \exp(i2k_n s^2) = 0,
\]

for \( s \neq 0 \). Equation (11) and (13) are chirped functions in \( s \) which have the same as the \( g(x, y) \) except for the coefficient and the fundamental spatial frequency. Hence, a lot of fluctuations are predicted for large \( s \), or large incident angles, and small fluctuations over small incident angles. Figure 3 shows the contrast calculated over a range of incident angles using an amplitude Fresnel zone plate with an innermost zone radius of \( \sim 16 \mu m \). The power is also shown in Figure 3(b). Note the correspondence of the locations at which extrema of the power and peaks of contrast are observed. Also, high fluctuations for large incident angles and small fluctuations for small incident angles are shown. The small variations shown in the contrast maxima are due to the small variations in the \( D \).

**CONCLUSION AND FUTURE WORK**

The contrast variations with changing incident angle were investigated for the interference fringe metrology using a Fresnel zone plate. It was shown that the contrast varies rapidly for large incident angles and slowly for small incident angles. Also, the incident angles where the peak and valley contrasts are observed can be predicted. It is desirable to have relatively high and uniform contrast in a wide range of incident angles, especially for the use of the VP-SBIL. The design of novel diffractive structures which have a uniform contrast variation are underway.

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