Introduction

Experiments have repeatedly demonstrated that vibration assistance can make cutting forces decrease. For instance, in grinding tests performed under dry conditions, the average force decreased by 15% when 3 kHz vibration was introduced [1]. Some research has been done to explain this phenomenon. Astashev, for example, developed a force model that includes the effect of vibration parallel to the cutting direction [2]. His model explains how forces reduce for a workpiece material whose strength is independent of cutting speed. The model predicts force reduction as a function of the ratio of maximum vibration speed to cutting speed. A large ratio leads to more force reduction. When the ratio is less than one (when cutting velocity exceeds maximum vibration velocity), vibration no longer affects the cutting force. Nazarenko proposed a theoretical model for friction behavior in the presence of mechanical vibration at various angles to motion direction [3]. If the vibration is parallel to the sliding direction, the equations for predicting force reduction are similar to Astashev’s. For vibrations normal and transverse to the sliding direction, friction force would also decrease.

Our research focuses on vertical vibration (normal to the cutting direction). Astashev’s mechanism does not apply in this case. However, we will make use of Nazarenko’s model when considering friction between the chip and tool rake face.

Figure 1 shows a 2-dimensional view of the orthogonal single point cutting process. Key parameters include depth of cut ($t_o$), chip thickness ($t_c$), cutting speed ($V$), chip velocity ($V_c$), shear angle ($\phi$) and rake angle ($\alpha$). Some of these parameters are prescribed: depth of cut, cutting speed, and rake angle. Shear angle, chip thickness, and chip velocity cannot be prescribed directly. They depend on material properties and rake face friction as well as $t_o$, $V$, and $\alpha$.

Introducing high frequency vibration in the vertical direction (applied to tool or workpiece) causes all these parameters to change during the machining process. Depth of cut varies in a straightforward way. The relative cutting speed between tool and workpiece changes in amplitude and direction. If the rake angle is defined using a reference perpendicular to the cutting direction, then it too changes continuously with vibration. So, $t_o$, $V$, and $\alpha$ modulate...
continuously about their nominal values. Changes in these parameters, in turn, influence the shear angle $\phi$, the chip thickness $t_o$, and the chip velocity $V_c$. Finally, rake face friction is influenced by the ratio of chip velocity to tool velocity. Since shear angle depends on rake face friction, the relationship amongst all the variables appears quite complex. Figure 2 depicts a map of the various factors and their interrelationships.

The shear angle plays a key role in determining forces in single point cutting; however, it is difficult to predict even in the simple case of no vibration assistance. Of the various models devised for predicting shear angle [4,5], none works well for all cases.

**Simplified Force Model**

Our first step in modeling how vibration influences force is to make a simple assumption about how vibration affects the shear angle. We assume that the shear plane angle with respect to a fixed coordinate system remains constant and assume the shear angle with respect to the cutting direction varies. Figure 3(a) shows a snapshot of the cutting geometry when the tool is at the top or bottom of its vibration cycle. In this case, the cutting direction is horizontal, and the shear angle is $\phi_1$. Figure 3(b) shows a snapshot when the tool is at its maximum vibration velocity. The instantaneous cutting direction rotates up, and the shear angle $\phi_2$ is less than $\phi_1$.

The instantaneous shear angle using this assumption is:

$$\phi_2 = \phi_1 - \arctan\left(\frac{V_v}{V}\right) \quad \text{where} \quad V_v = a \omega \sin \omega t$$

(1)
Suppose $V=1$ m/s, $\phi_1=20^\circ$, and $\alpha=0^\circ$. Fig. 4 shows how $\phi_2$ changes with time for a couple of different vibration conditions.

![Graph showing shear angle variation](image)

**Fig. 4:** Shear angle variation ($V=1$ m/s, $\phi_1=20^\circ$, and $\alpha=0^\circ$)

### Chip Velocity and Average Friction Coefficient

Based on our prediction for shear angle, we can now calculate how the chip velocity modulates. Then we can compare the chip velocity to the vibration velocity of the tool rake face. The chip velocity can be calculated from:

$$V_c = V \cdot \frac{t_o}{t_c} = V \cdot \frac{\sin \phi}{\cos(\phi - \alpha)} \quad (2)$$

![Graph comparing $V_a$ and $V_c$](image)

**Fig. 5:** Comparison of $V_a$ (bold line) and $V_c$

For the case of $V=1$ m/s, $\phi_1=20^\circ$, $\alpha=0^\circ$, $\omega=5000$ rad/s, and $a=40$ $\mu$m, Fig. 5 shows the variation in chip velocity (based on Eqs 2 and 1) and vibration velocity. According to Fig. 5, the vibration
velocity exceeds the chip velocity for a short time during each vibration cycle. At these times, the friction force between the chip and tool changes direction. Consequently, when averaged over many cycles, the friction force is less than its nominal value.

Increasing the vibration frequency or amplitude increases the amount of time that the vibration speed exceeds the chip velocity and thus further reduces the average friction force. Fig. 6 shows the reduction in rake face friction force as maximum vibration velocity increases. If the vibration velocity can be made high enough, significant reductions in friction force (and thus overall machining forces) are possible.

![Fig.6: Reduction in rake face friction (note that an $\omega$ of 5000 rad/sec and an $a$ of 40 $\mu$m produces a maximum vibration velocity of 0.2 m/s)](image_url)

**Conclusions**

This model, based on a simple geometric assumption about the shear angle, predicts that moderately high vibration frequencies (thousands of rad/sec) can produce significant force reductions. However, further refinement of the shear angle assumption is necessary. Future work will investigate the effects of depth of cut and friction force variation on the shear angle. In addition, single point cutting experiments will be done to determine how the vibration frequency and amplitude actually affect the force on the rake face.

**References**


